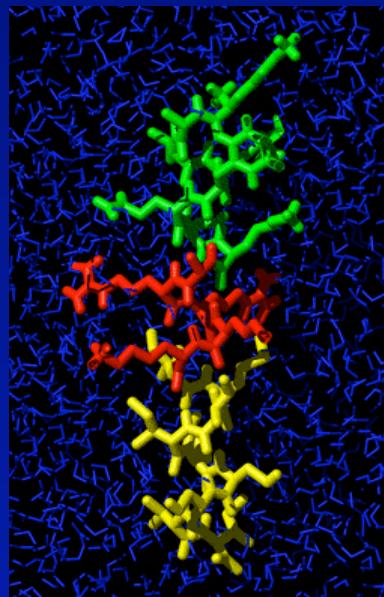
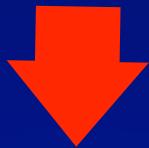
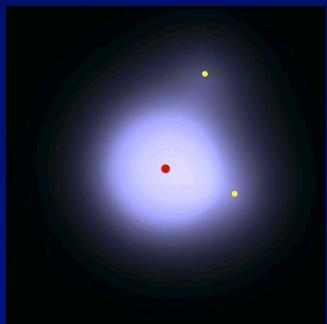


Molecular Dynamics Simulations



Schrödinger equation

$$\hat{H}\phi(\vec{r}, \vec{R}) = E\phi(\vec{r}, \vec{R})$$

Born-Oppenheimer approximation

$$\hat{H}_e\phi(\vec{r}; \vec{R}) = E_e(\vec{R})\phi_e(\vec{r}; \vec{R})$$

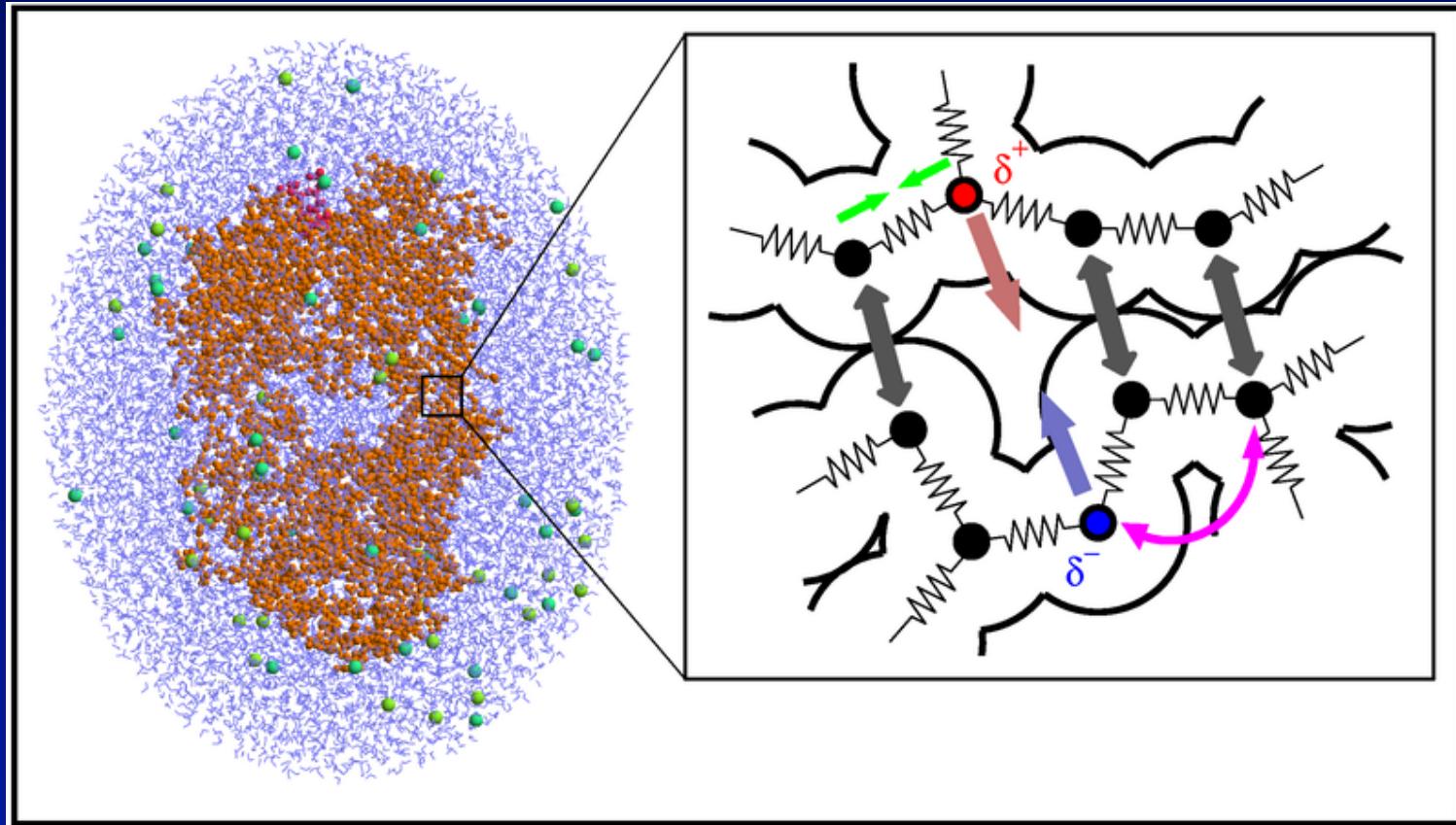
Nucleic motion described classically

$$m_i \frac{d^2}{dt^2} \vec{R}_i = -\vec{\nabla}_i E(\vec{R})$$

Empirical Force field

$$E(\vec{R}) = \sum_{\text{bonded}} E_i(\vec{R}) + \sum_{\text{non-bonded}} E_i(\vec{R})$$

Molecular Dynamics Simulations



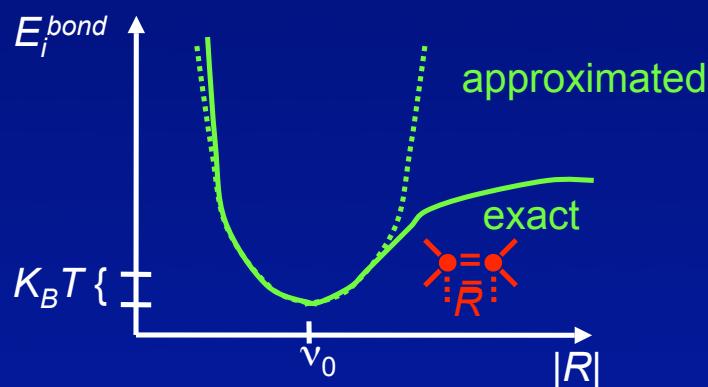
Interatomic interactions

Molecular dynamics-(MD) simulations of Biopolymers

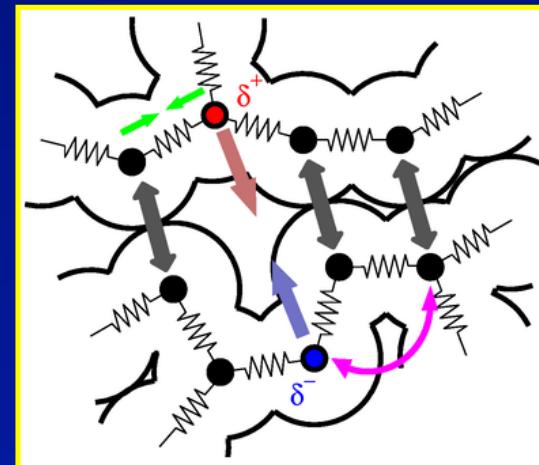
- Motions of nuclei are described classically, $(*) \quad m_a \frac{d^2}{dt^2} \mathbf{R}_\alpha = -\nabla_\alpha E_{el}(\mathbf{R}_1, \dots, \mathbf{R}_N), \quad \alpha = 1, \dots, N.$
- Potential function E_{el} describes the electronic influence on motions of the nuclei and is approximated empirically → „classical MD“:

$$E_{el} \approx \sum_{\text{Bindungen } i} E_i^{\text{bond}} + \sum_{\text{Bindungs-winkel } j} E_j^{\text{angle}} + \sum_{\text{Dihedral-winkel } k} E_k^{\text{dihed}} + \sum (\text{ } E_{\alpha, \beta}^{\text{Coul.}} + E_{\alpha, \beta}^{\text{rep.}} + E_{\alpha, \beta}^{\text{vdW}}) + \dots,$$

Covalent bonds



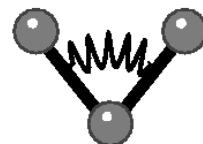
Non-bonded interactions



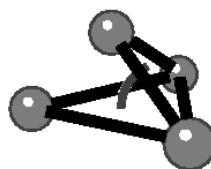
„Force-Field“



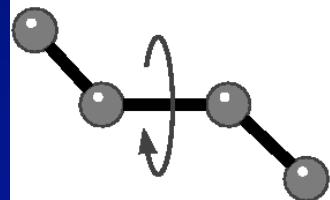
$$V_B = \sum_{\text{Bindungen}} \frac{1}{2} K_b (b - b_0)^2$$



$$V_a = \sum_{\text{Winkel}} \frac{1}{2} K_\theta (\theta - \theta_0)^2$$



$$V_{imp} = \sum_{\text{Extraplanarwinkel}} \frac{1}{2} K_\zeta (\zeta - \zeta_0)^2$$



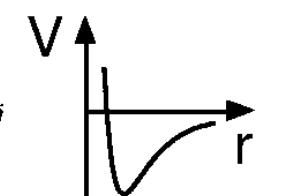
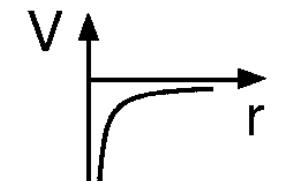
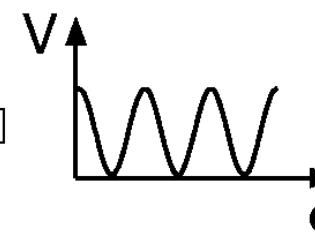
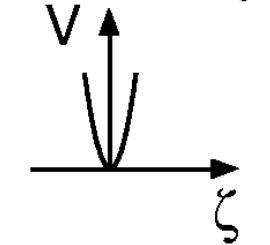
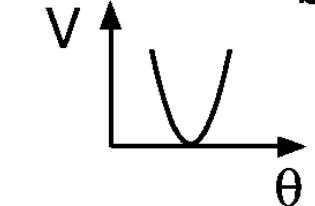
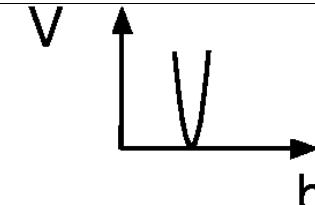
$$V_D = \sum_{\text{Dihedralwinkel}} K_\varphi [1 + \cos(n\varphi - \delta)]$$



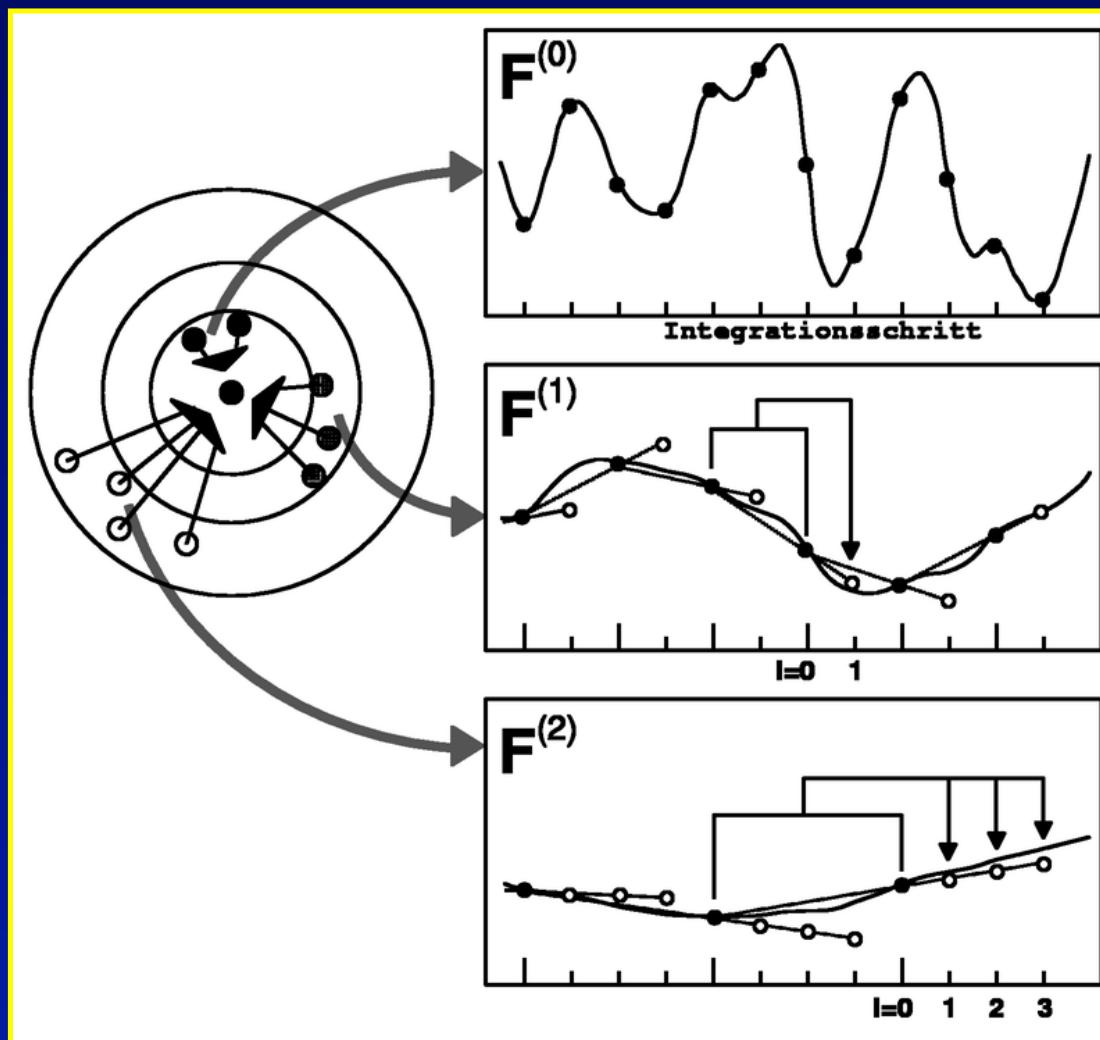
$$V_q = \sum_{\text{Paare(i,j)}} q_i q_j / (4\pi\epsilon_0\epsilon_r r_{ij})$$

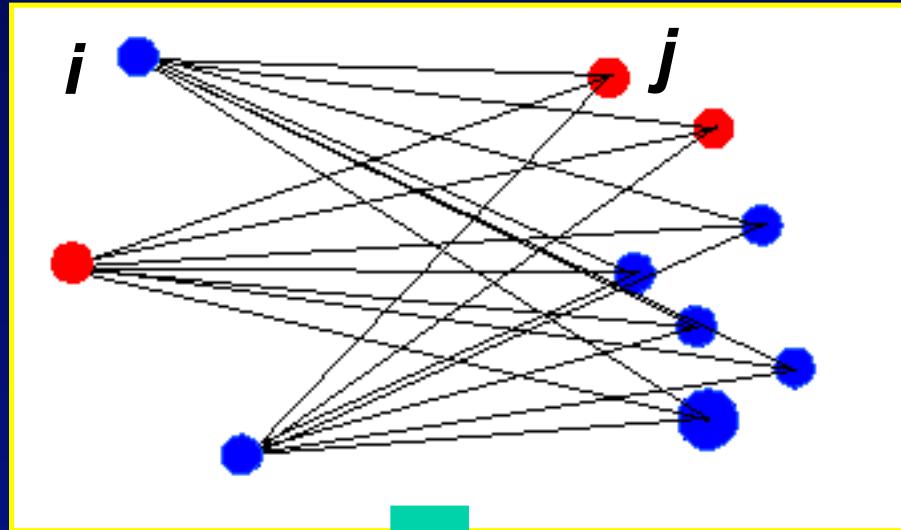


$$V_{vdW} = \sum_{\text{Paare(i,j)}} C_{12}(i,j)/r_{ij}^{12} - C_6(i,j)/r_{ij}^6$$



Multiple Time Stepping

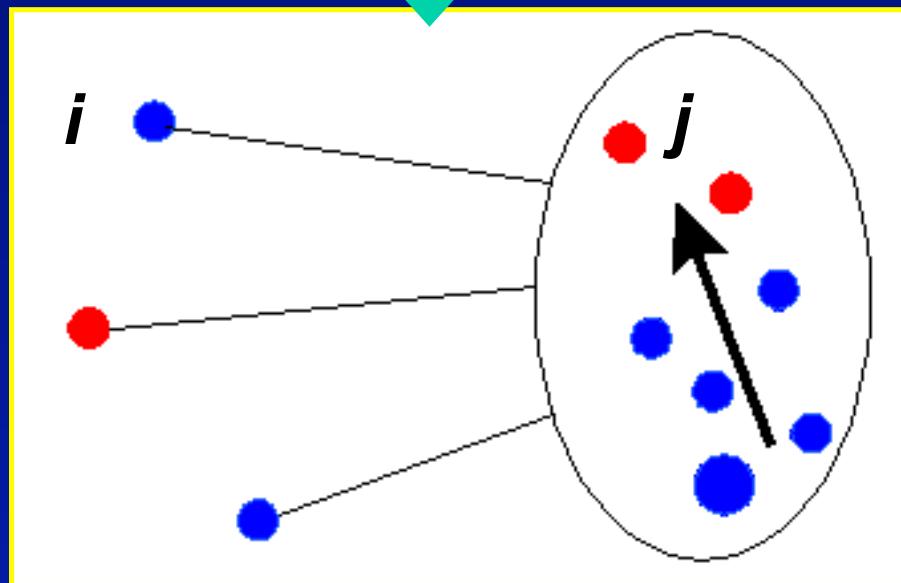




Multipole Methods

$O(N^2)$

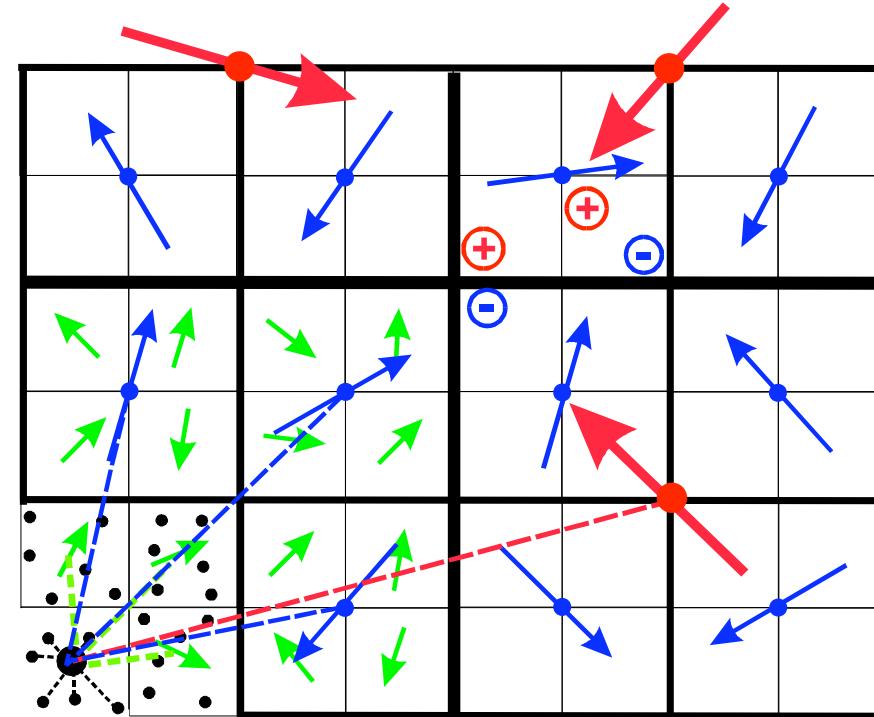
1. Taylor expansion



Exact for infinite
multipole series

Fast Multipole Method (FMM)

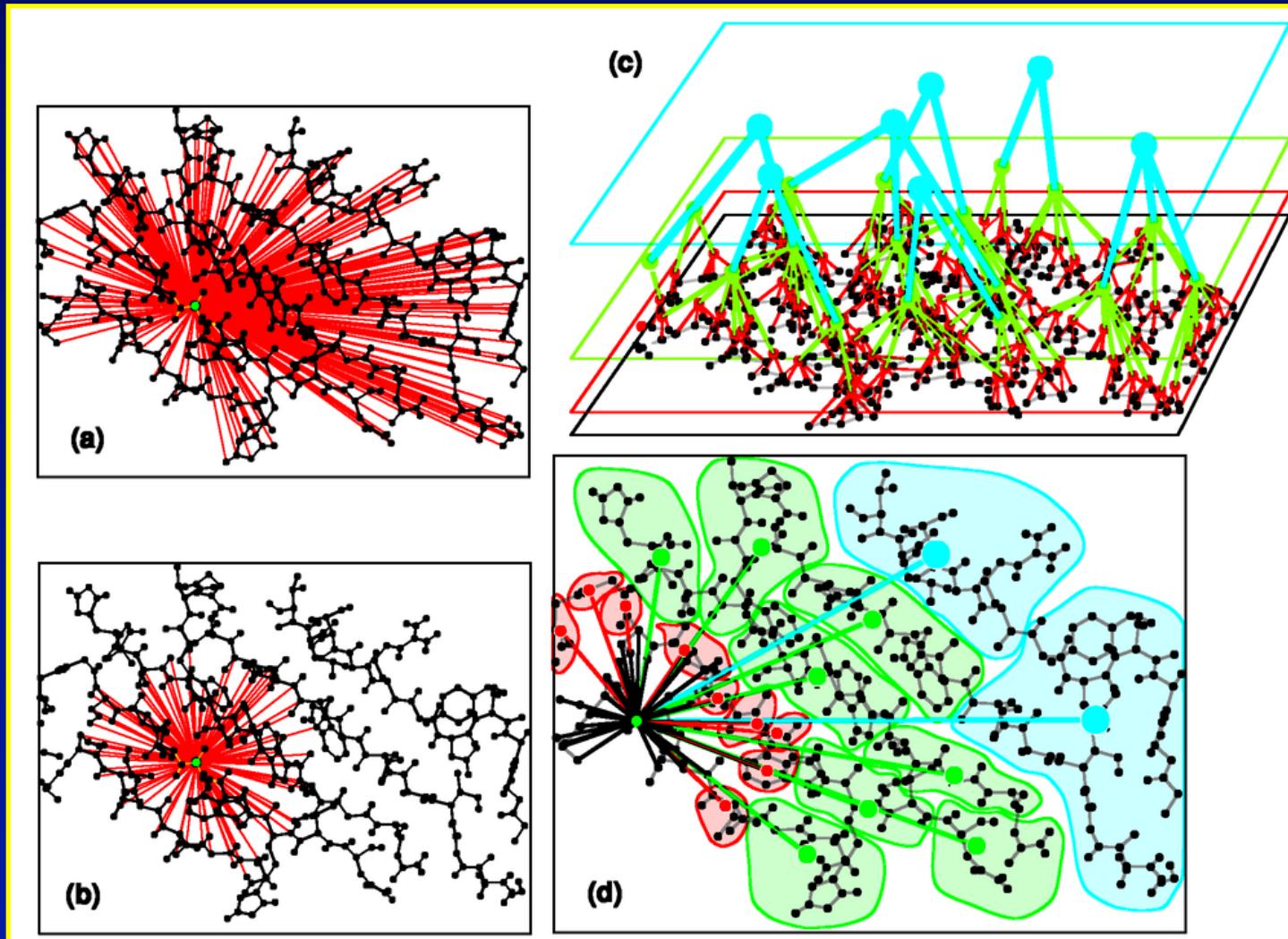
$\rightarrow O(N)$



+ arbitrary accuracy

- high order expansions required
to achieve moderate accuracy

Fast structure-adapted multipole methods: O(N)



Molecular Dynamics Simulation

Molecule: (classical) N-particle system

Newtonian equations of motion: $m_i \frac{d^2}{dt^2} \vec{r}_i = \vec{F}_i(\bar{r})$

$$\vec{F}_i(\bar{r}) = -\nabla_i V(\bar{r})$$

with $\bar{r} = (\vec{r}_1, \dots, \vec{r}_N)$

Integrate numerically via the „leapfrog“ scheme:

$$\begin{aligned} \boldsymbol{v}(t + \frac{\Delta t}{2}) &= \boldsymbol{v}(t - \frac{\Delta t}{2}) + \frac{\boldsymbol{F}(t)}{m} \Delta t && \text{with} \\ \boldsymbol{r}(t + \Delta t) &= \boldsymbol{r}(t) + \boldsymbol{v}(t + \frac{\Delta t}{2}) \Delta t && \Delta t \approx 1 \text{ fs!} \end{aligned}$$

(equivalent to the Verlet algorithm)

Computational task:

Solve the Newtonian equations of motion:

$$m_i \frac{d^2}{dt^2} \vec{x}_i = \vec{F}_i(\vec{x}_1, \dots, \vec{x}_N) \quad i = 1 \dots N$$

$$\vec{F}_i(\vec{x}_1, \dots, \vec{x}_N) = -\nabla_i E(\vec{x}_1, \dots, \vec{x}_N)$$

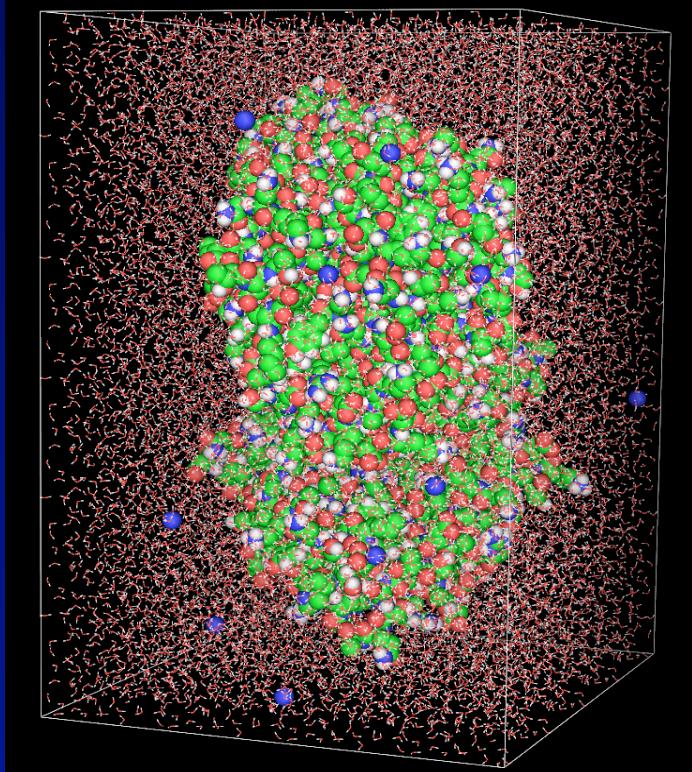
$$E(\dots) = E_{\text{bond}}(\dots) + E_{\text{non-bond}}(\dots) + E_{\text{ext.}}(\dots)$$

$$\sum_{\text{bonds } ij} k_{ij} (r_{ij} - r_{ij}^{\text{eq}})^2 \quad \sum_{\text{pairs } ij} \frac{q_i q_j}{4\pi\varepsilon r_{ij}}$$

$$O(N)$$

$$O(N^2)$$

Role of environment - solvent



box

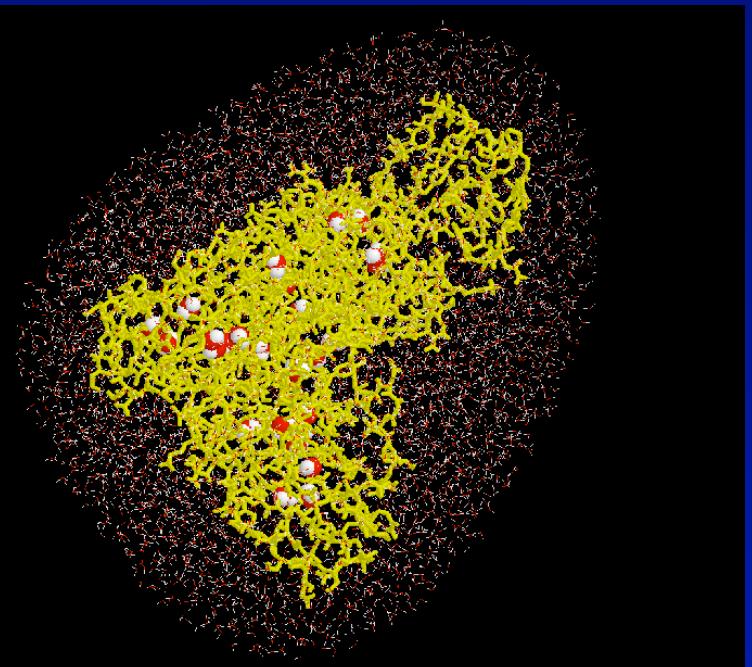
or

droplet?

explicit

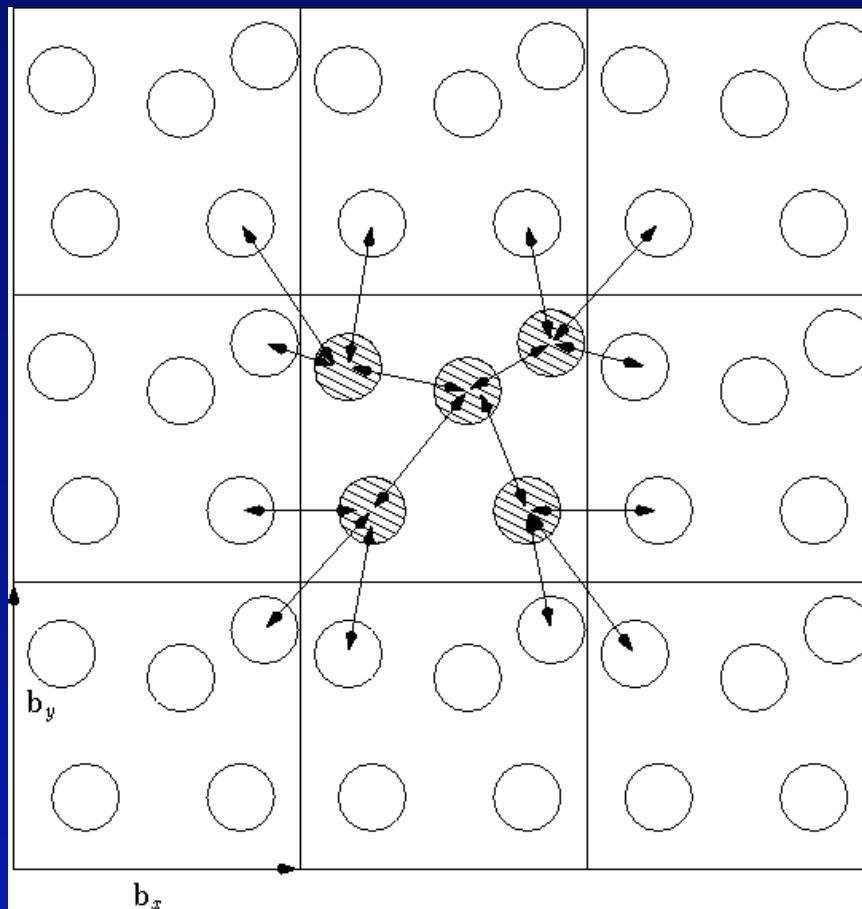
or

implicit?

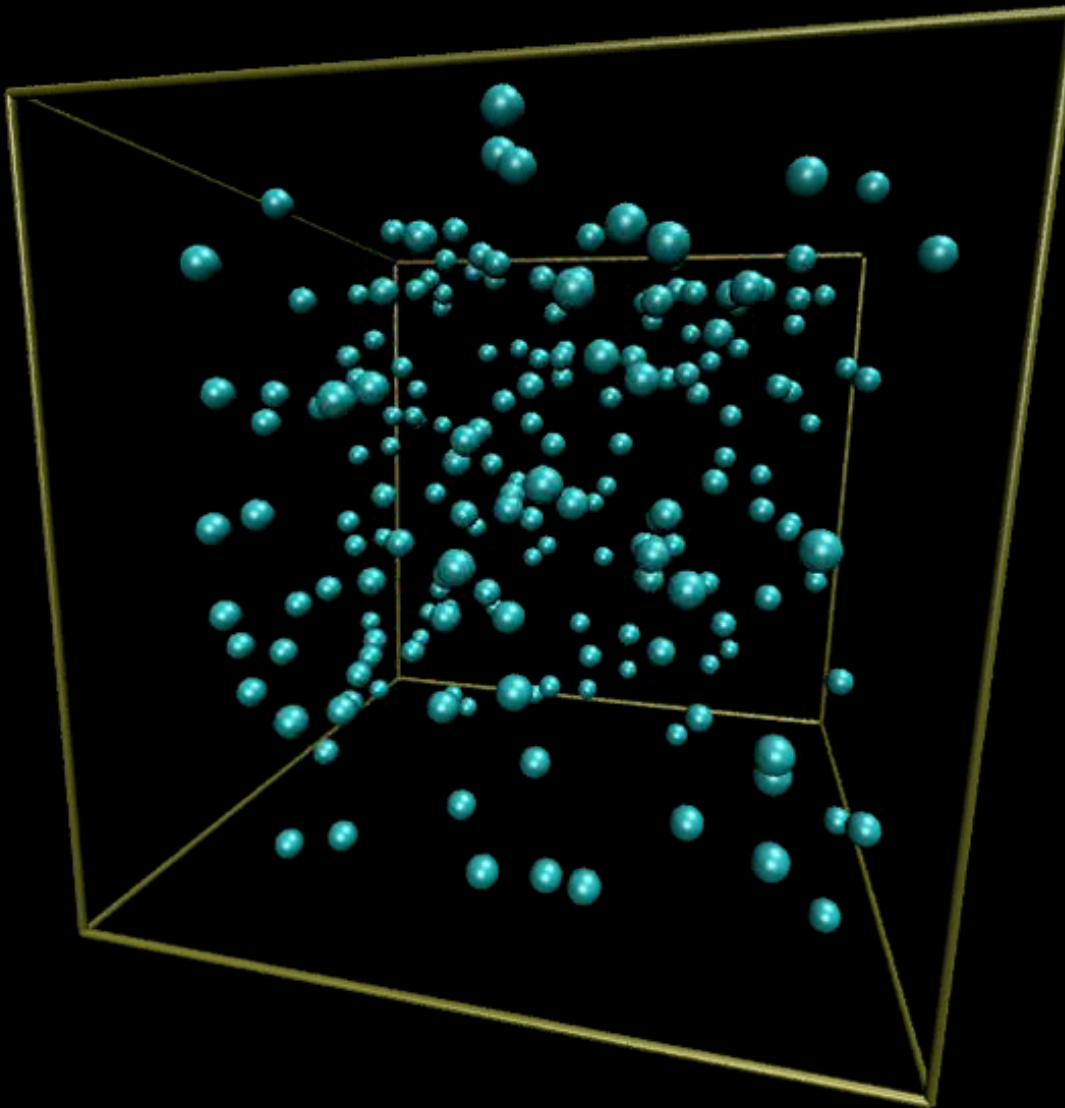


Surface (tension) effects?

periodic boundary conditions
and the minimum image convention

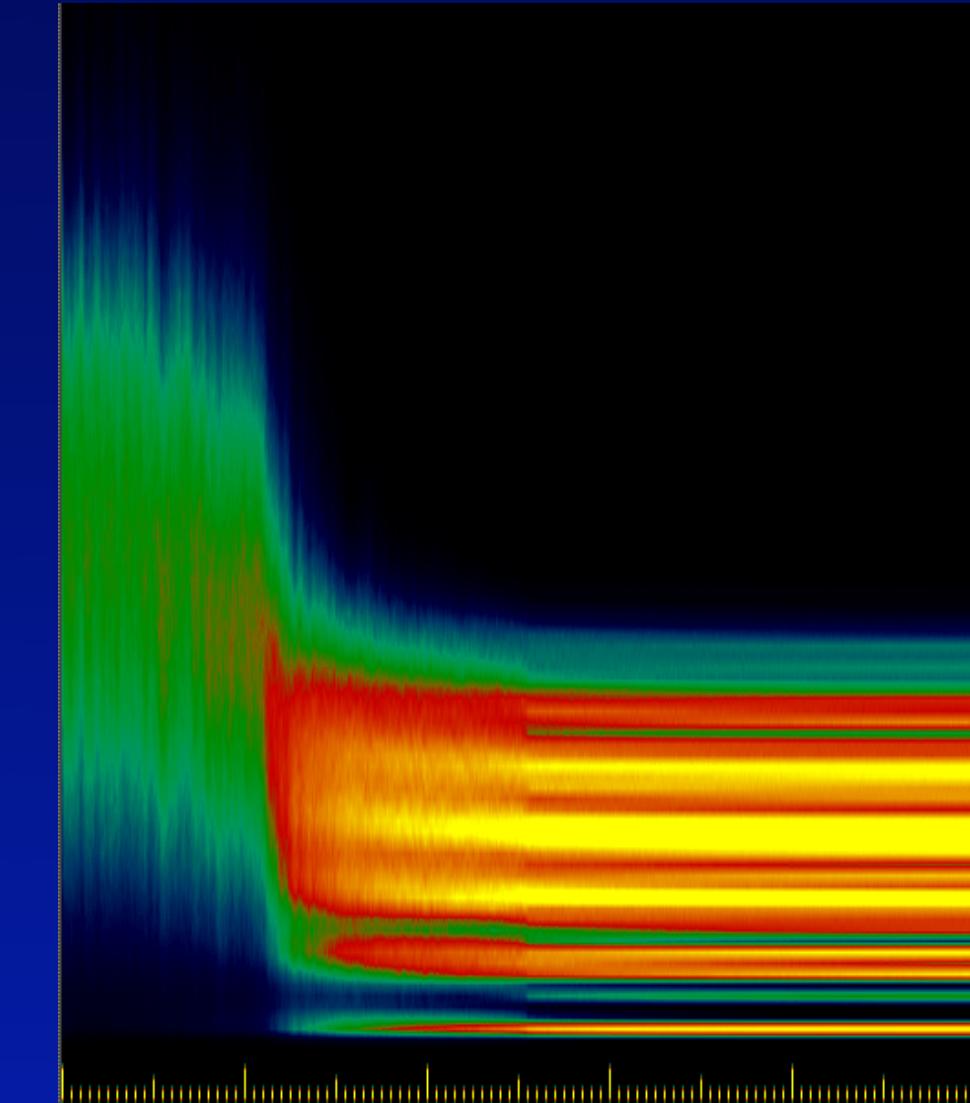


MD-Experiments with Argon Gas



Radial distribution function

distance

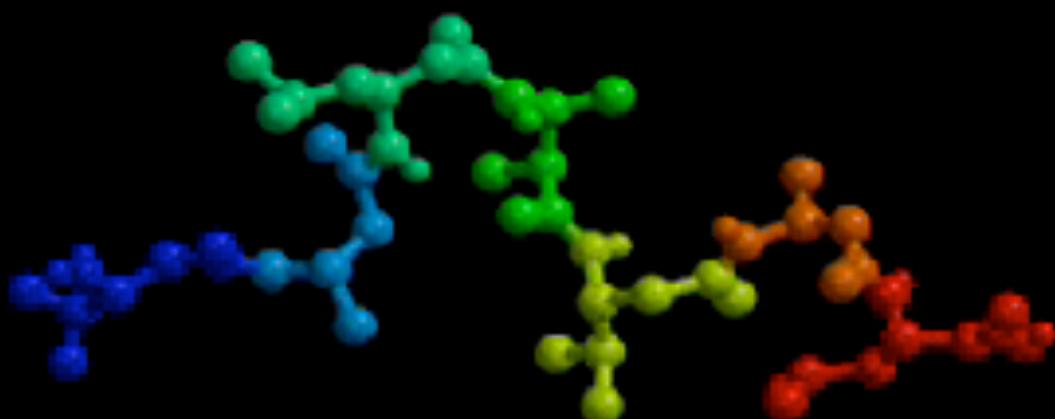


300 K

70 K

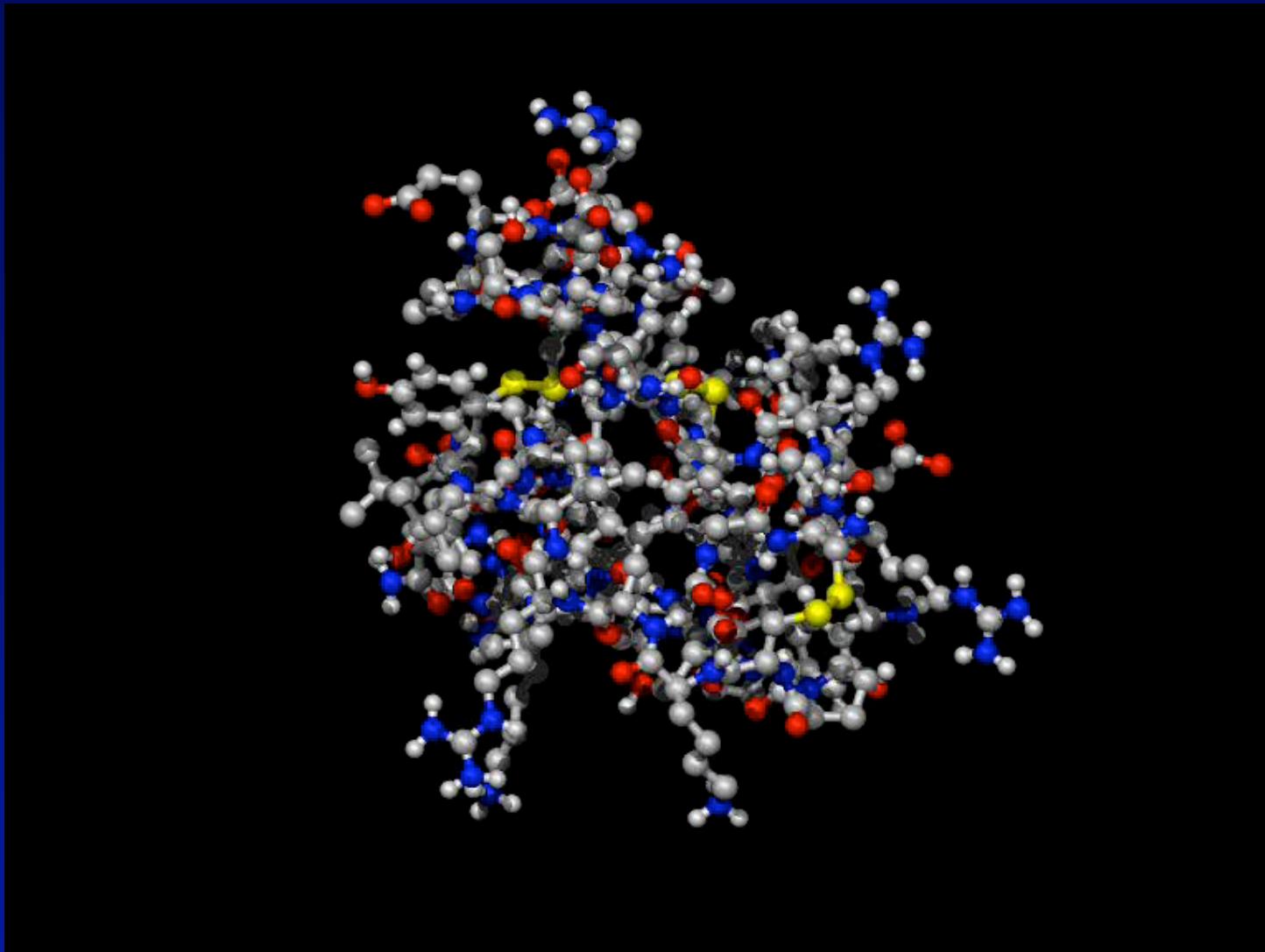
10 K

Reversible Folding Dynamics of a β -Peptide

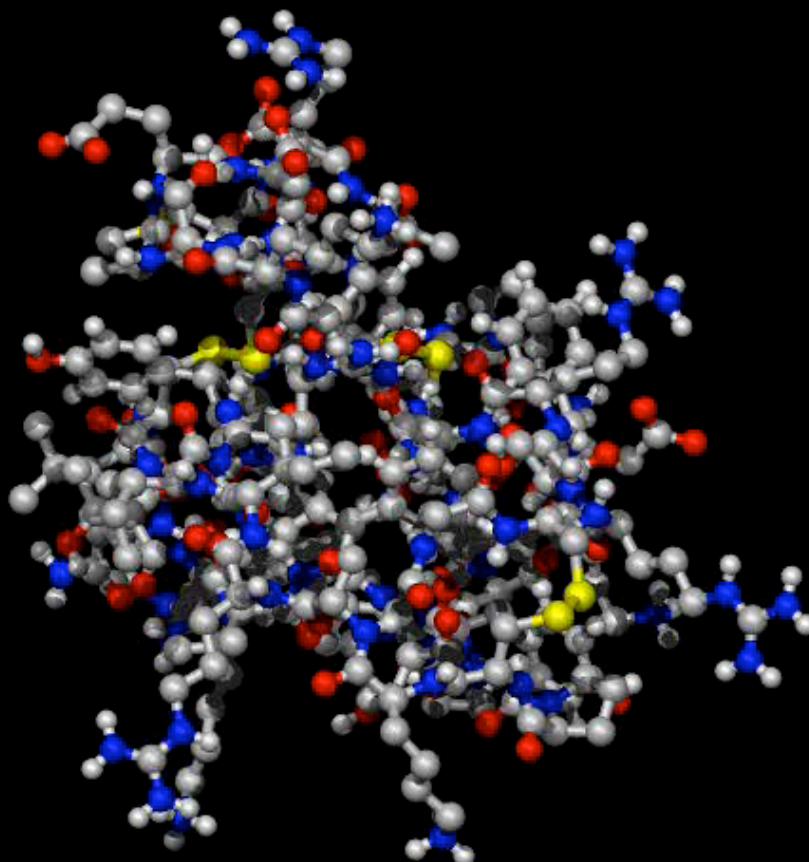


X. Daura, B. Jaun, D. Seebach, W.F. van Gunsteren, A.E. Mark, *J. Mol. Biol.* **280** (1998) 925

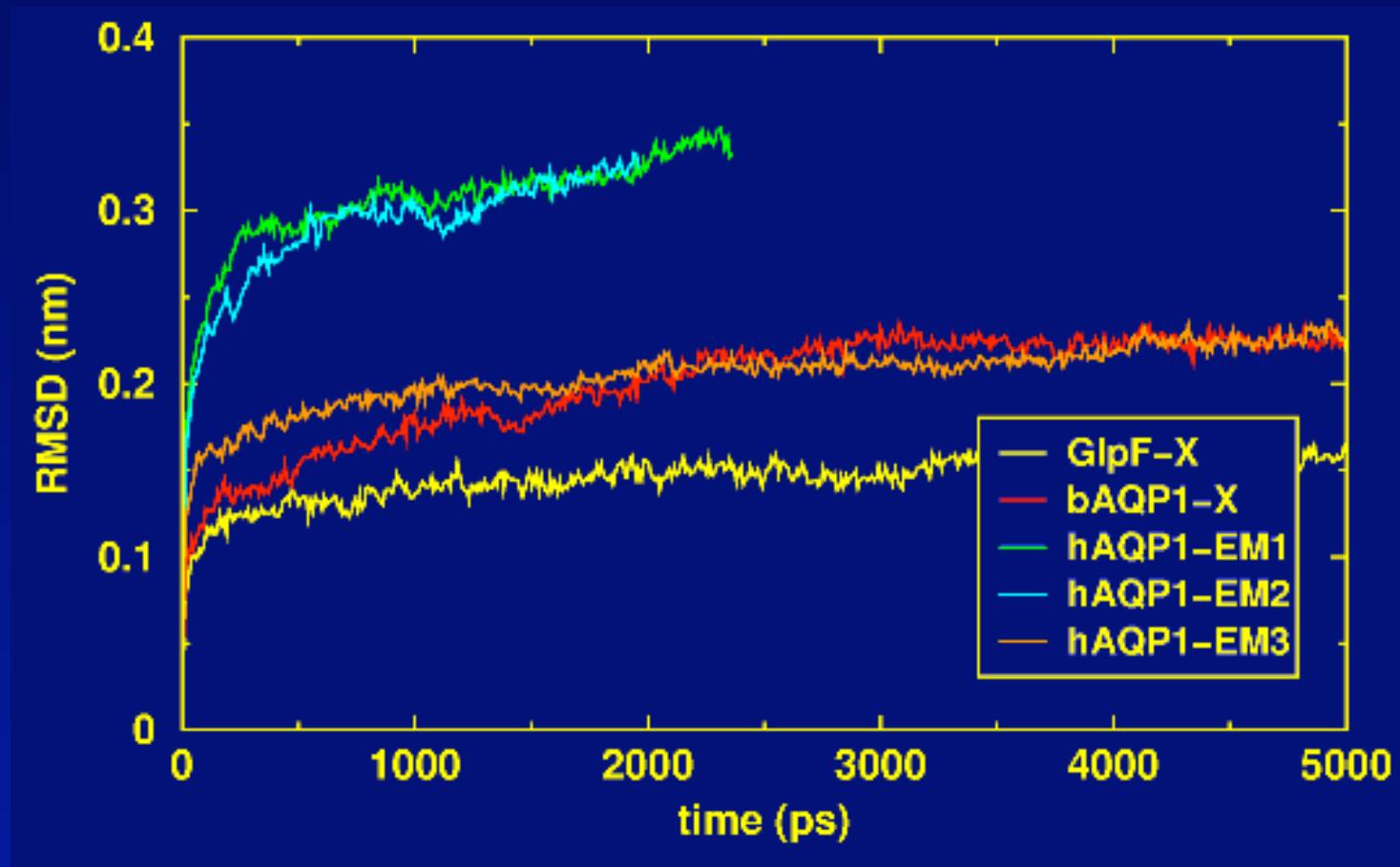
BPTI: Minimization



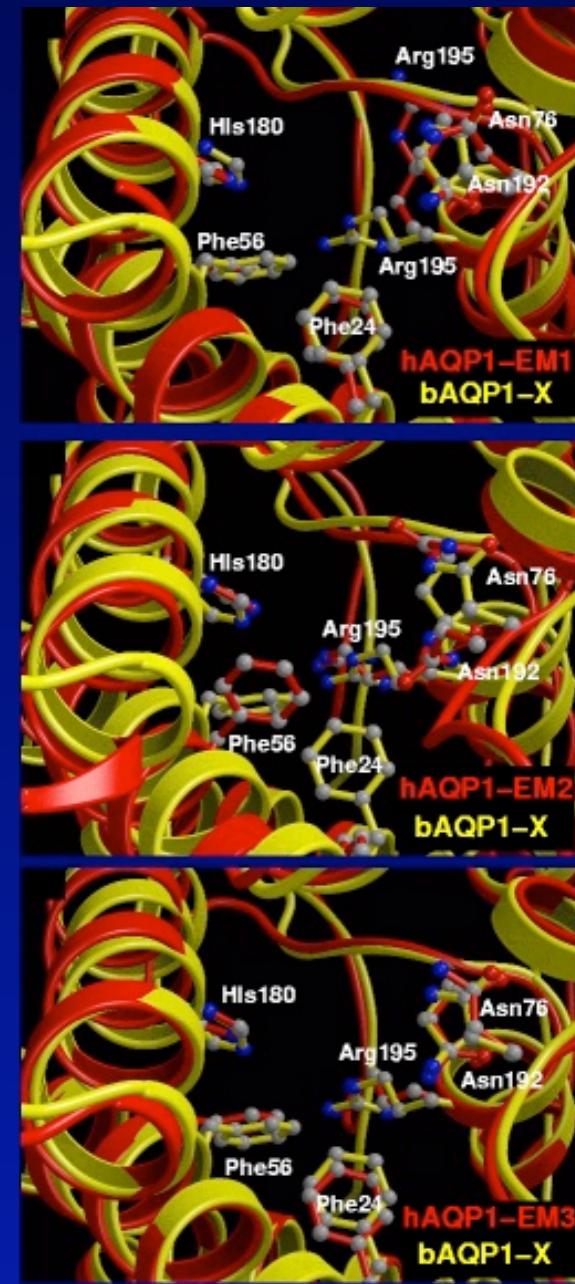
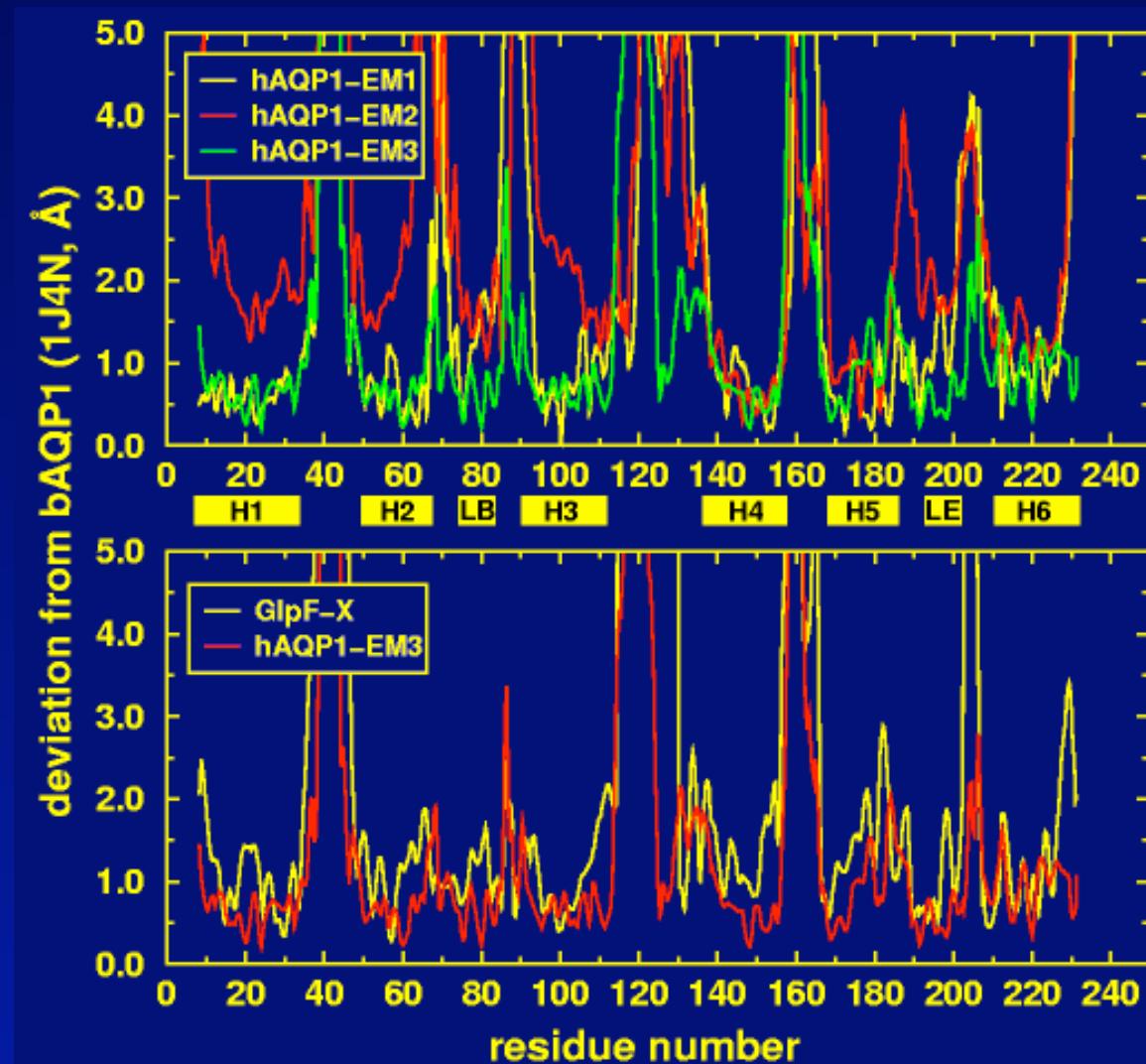
BPTI: Molecular Dynamics (300K)



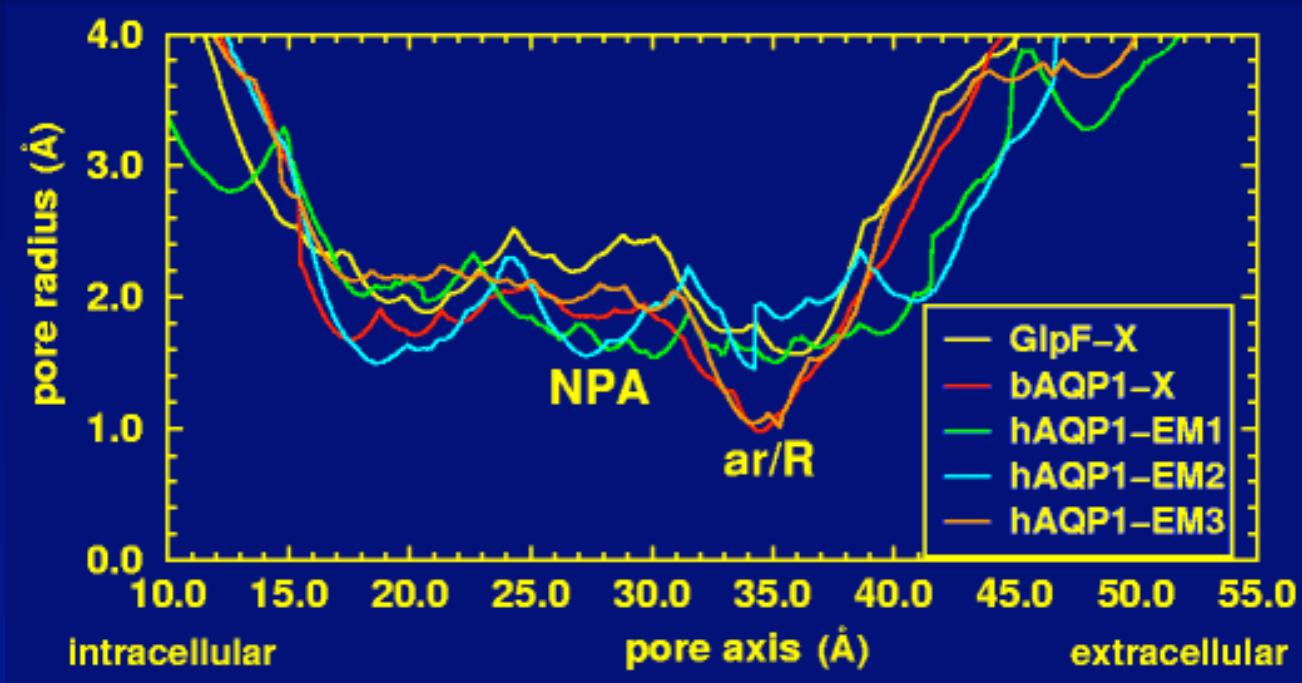
Stability in MD simulations as a structure quality indicator



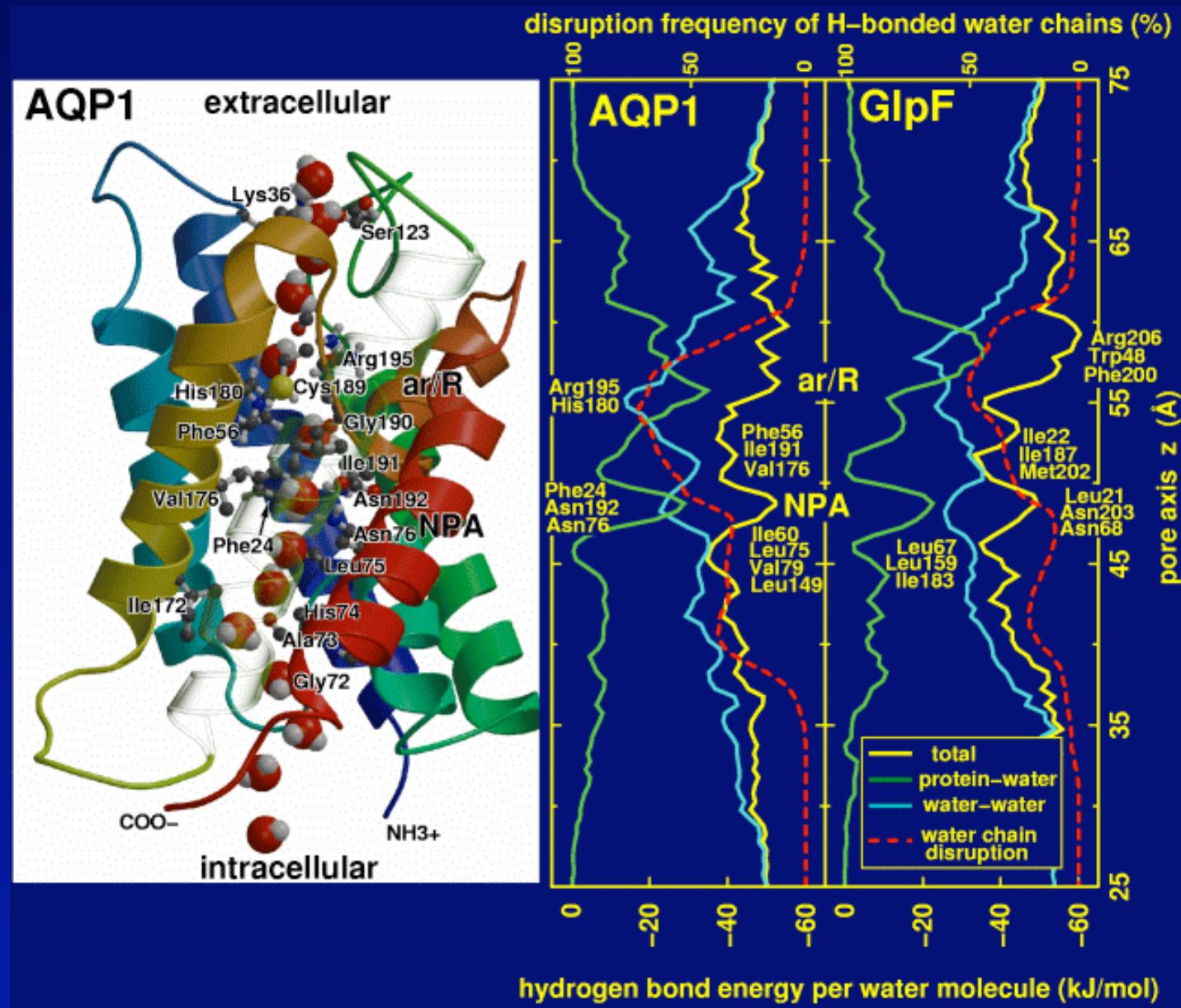
bAQP1 EM per residue deviations from x-ray



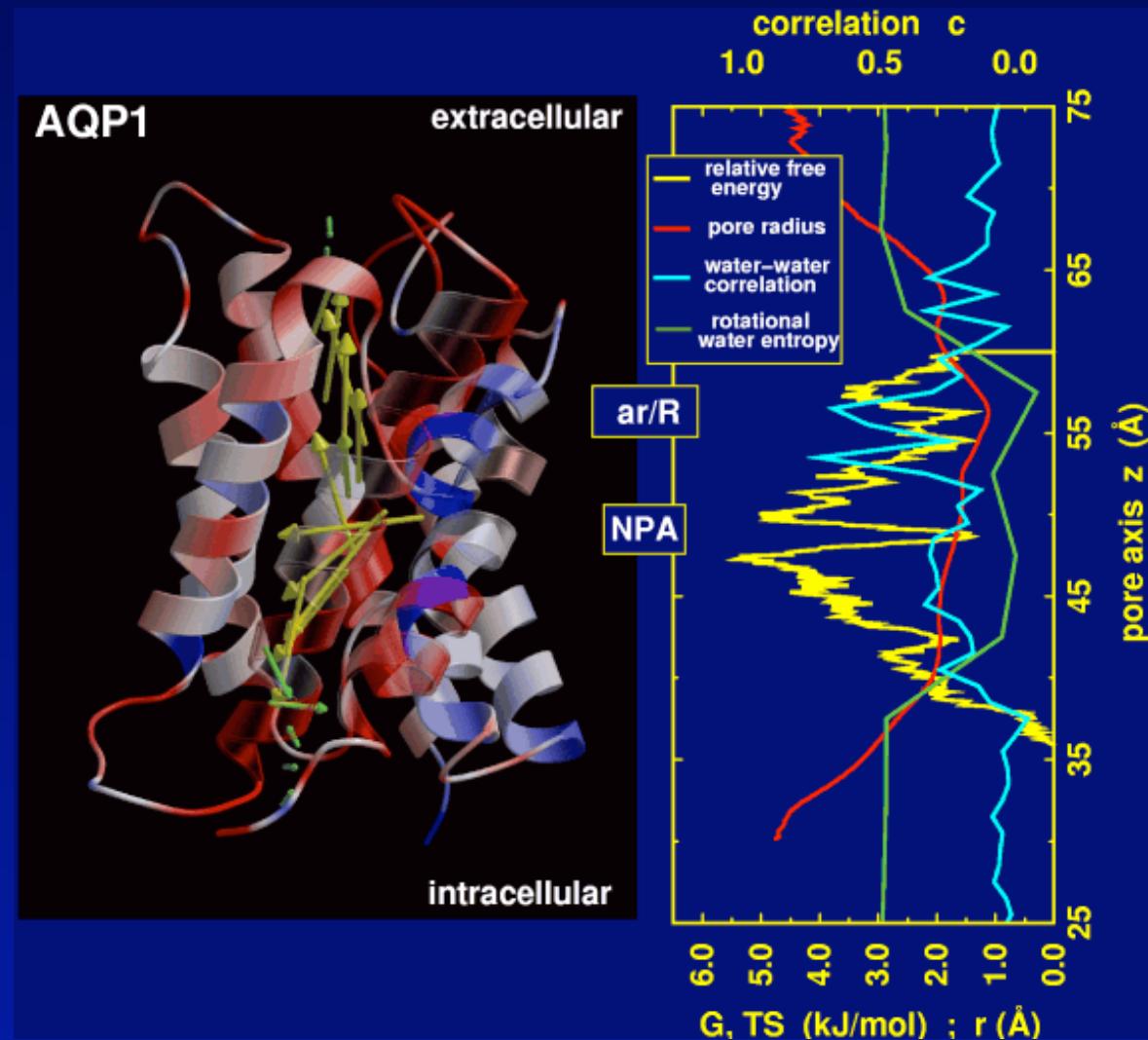
Comparison of pore radius profiles



Water pathway and hydrogen bonding in Aquaporin-1

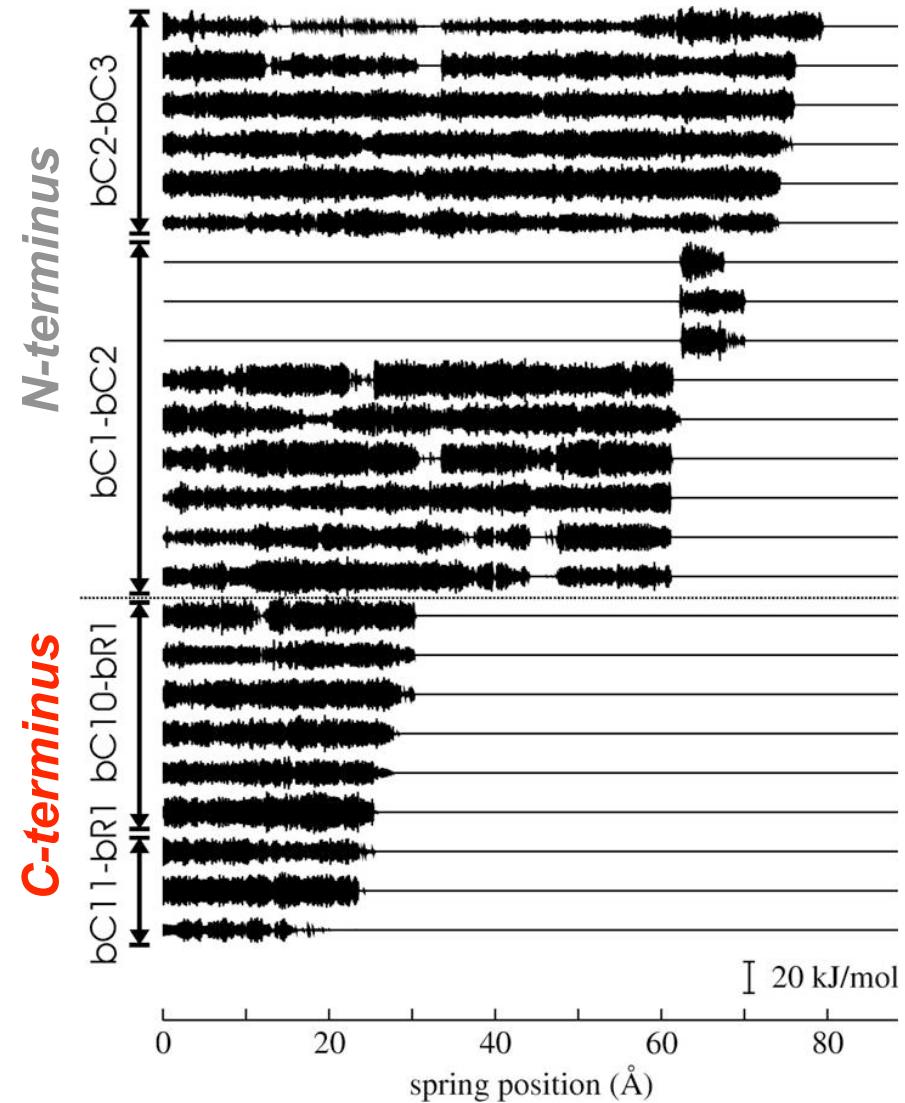


Choreography of water molecules in Aquaporin-1



B.L. de Groot, H. Grubmüller, *Science* **294**, 2353 (2001)

Asymmetric rupture of the terminal beta sheets



N

C

