

## THE RENORMALIZABILITY OF VECTOR MESON INTERACTIONS

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**Abstract:** A new and less restrictive criterion is deduced for the renormalizability of vector meson interactions. As an example, the electromagnetic interaction of a charged vector meson with unit anomalous magnetic moment is shown to be renormalizable. In order to construct a renormalizable theory of the elementary-particle decays in which such a particle plays the role of an intermediary, it is found necessary to assume an intimate connection between decay-inducing and electrodynamic interactions.

### 1. Kinematical Considerations

Recent advances in the study of elementary-particle decays have encouraged speculation concerning the nature of the interactions responsible for these processes. In particular, the hypothesis of an unstable vector meson whose universal interactions underlie all such decays has recently achieved some favour <sup>1</sup>). Those to whom renormalizability seems a desirable attribute of a quantum-field theory may object to such a theory because of the alleged unrenormalizability of most vector-meson couplings <sup>2</sup>). As a partial answer we shall deduce a new criterion which admits as renormalizable many interactions (among them the electrodynamics of an isolated charged vector field) that have not until now been so regarded. A suggestive connection between this renormalizability criterion and a certain limited symmetry of the Lagrange function may lead us to a renormalizable theory of a charged vector meson with appropriate decay-inducing couplings.

We must first evaluate several Green's functions associated with the propagation of a free vector field. If we are to use the formalism of Schwinger <sup>3</sup>), in which a variational principle yields both field equations and commutation relations, the Lagrange function must contain only first derivatives and therefore we must introduce dynamical variables  $Z_\mu$  and  $Z^{\mu\nu}$  which satisfy first order differential equations. The Lagrange function,

$$L = -\frac{1}{2} \left[ \frac{1}{2} Z^{\mu\nu} (\partial_\mu Z_\nu - \partial_\nu Z_\mu) - Z_\nu \partial_\mu Z^{\mu\nu} - \frac{1}{2} Z^{\mu\nu} Z_{\mu\nu} + M^2 Z^\mu Z_\mu \right] + \left[ \frac{1}{2} m_{\mu\nu} Z^{\mu\nu} - j_\mu Z^\mu \right], \quad (1.1)$$

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in which  $M$  is the mass of the vector meson, includes couplings to an external vector source,  $j_\mu$ , and to an external antisymmetric-tensor source,  $m_{\mu\nu}$ . The above form of the tensor coupling is chosen rather than the similar expression,  $m^{\mu\nu}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)$ , for two reasons. After an integration by parts the latter expression becomes identical to the vector coupling of  $Z_\mu$  to an effective current  $\partial_\lambda m^{\lambda\nu}$ . Moreover, to include explicit derivatives in the interaction Lagrangian would destroy the canonical form of the commutation relations implied by  $L$ .

The first order field equations following from (1.1) are

$$\begin{aligned} Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu + m_{\mu\nu} \\ \partial_\mu Z^{\mu\nu} &= M^2 Z^\nu + j^\nu. \end{aligned} \quad (1.2)$$

These may be combined to form second order equations for  $Z_\mu$ ,

$$(\not{p}^2 + M^2)Z_\mu = - \left( \delta_\mu^\nu + \frac{\not{p}_\mu \not{p}_\nu}{M^2} \right) j_\nu + i \not{p}^\lambda m_{\lambda\mu}, \quad (1.3a)$$

and for  $Z_{\mu\nu}$ ,

$$(\not{p}^2 + M^2)Z_{\mu\nu} = -i(\not{p}_\mu j_\nu - \not{p}_\nu j_\mu) + (\not{p}_\mu \not{p}^\lambda m_{\lambda\nu} - \not{p}_\nu \not{p}^\lambda m_{\lambda\mu}) - (\not{p}^2 + M^2)m_{\mu\nu}. \quad (1.3b)$$

With the connections

$$\mathcal{G}_\nu^\mu(x-x') = \frac{\delta Z^\mu(x)}{\delta j_\nu(x')}, \quad (1.4a)$$

$$\mathcal{G}_\lambda^{\mu\nu}(x-x') = \frac{\delta Z^{\mu\nu}(x)}{\delta j_\lambda(x')} = - \frac{\delta Z_\lambda(x')}{\delta m^{\mu\nu}(x)}, \quad (1.4b)$$

$$\mathcal{G}_{\lambda\sigma}^{\mu\nu}(x-x') = \frac{\delta Z^{\mu\nu}(x)}{\delta m_{\lambda\sigma}(x')}, \quad (1.4c)$$

the vector meson propagators are identified as

$$\mathcal{G}_\nu^\mu = - \left( \delta_\nu^\mu + \frac{\not{p}_\nu \not{p}_\mu}{M^2} \right) \frac{1}{\not{p}^2 + M^2}, \quad (1.5a)$$

$$\mathcal{G}_\lambda^{\mu\nu} = -\mathcal{G}_{\lambda\nu}^{\mu\sigma} = -i(\not{p}^\mu \delta_\lambda^\nu - \not{p}^\nu \delta_\lambda^\mu) \frac{1}{\not{p}^2 + M^2}, \quad (1.5b)$$

$$\mathcal{G}_{\lambda\sigma}^{\mu\nu} = (\not{p}^\mu \not{p}_\lambda \delta_\sigma^\nu - \not{p}^\mu \not{p}_\sigma \delta_\lambda^\nu + \not{p}^\nu \not{p}_\sigma \delta_\lambda^\mu - \not{p}^\nu \not{p}_\lambda \delta_\sigma^\mu) \frac{1}{\not{p}^2 + M^2} - (\delta_\lambda^\mu \delta_\sigma^\nu - \delta_\sigma^\mu \delta_\lambda^\nu). \quad (1.5c)$$

Elimination of the vector meson field from the Lagrange function leads to an induced interaction among the currents to which  $Z$  is coupled. For  $m^2 \gg -\not{p}^2$  the leading terms are  $M^{-2}j^\mu j_\mu - m^{\mu\nu} m_{\mu\nu}$  and, consequently, the

vector meson can generate direct A, V and T couplings. More relevantly we note that in the limit of large momenta both (1.5a) and (1.5c) are of order one, while (1.5b) decreases with  $p^{-1}$ .

By similar means, the electrodynamic propagators in Lorentz gauge are found to be

$$\mathcal{D}_\nu^\mu = \delta_\nu^\mu p^{-2}, \quad (1.6a)$$

$$\mathcal{D}_\lambda^{\mu\nu} = -\mathcal{D}_\lambda^{\nu\mu} = -i(p^\nu \delta_\lambda^\mu - p^\mu \delta_\lambda^\nu) p^{-2}, \quad (1.6b)$$

$$\mathcal{D}_{\lambda\sigma}^{\mu\nu} = (p^\mu p_\lambda \delta_\sigma^\nu - p^\nu p_\lambda \delta_\sigma^\mu + p^\nu p_\sigma \delta_\lambda^\mu - p^\mu p_\sigma \delta_\lambda^\nu) p^{-2} - (\delta_\lambda^\mu \delta_\sigma^\nu - \delta_\sigma^\mu \delta_\lambda^\nu). \quad (1.6c)$$

With our choice of tensor coupling, (1.6c) satisfies  $\partial_\mu \mathcal{D}_{\lambda\sigma}^{\mu\nu} = 0$ .

Decomposing  $Z_\mu$  into transverse and longitudinal parts,

$$Z_\mu = Z_\mu^t + \partial_\mu A, \quad (1.7)$$

where

$$\partial^\mu Z_\mu^t = 0,$$

we deduce for the propagators of  $Z_\mu^t$  and  $A$  the expressions

$$\frac{\delta Z_\mu^t}{\delta j_\nu} = \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{M^2} \right) \frac{1}{p^2 + M^2}, \quad (1.8a)$$

$$\frac{\delta Z_\lambda^t}{\delta m^{\mu\nu}} = -\frac{\delta Z^{\mu\nu}}{\delta j_\lambda} = -i(p^\mu \delta_\lambda^\nu - p^\nu \delta_\lambda^\mu) \frac{1}{p^2 + M^2}, \quad (1.8b)$$

$$\frac{\delta A}{\delta j_\mu} = -\frac{p_\mu}{M^2 p^2}, \quad (1.9a)$$

$$\frac{\delta A}{\delta m_{\mu\nu}} = 0. \quad (1.9b)$$

## 2. Renormalization Criteria

We shall consider the theory of a vector meson interacting with other fields of spin not greater than one. Let the interaction Lagrange function be expressed in terms of  $Z_\mu$  and its explicit derivatives together with those fields to which it is coupled. In the sum

$$L = \sum_i g_i L_i(f, b, v, s), \quad (2.1)$$

each term is a product consisting of  $f$  factors of a Fermion field,  $b$  factors of either a spinless Boson field or the electrodynamic vector potential,  $v$  factors of  $Z_\mu$ , and  $s$  derivatives  $K(F_e, B_e, V_e, \{n_i\})$  is defined as the degree of divergence of a primitive contribution of order  $\prod_i (g_i)^{n_i}$  to a transition amplitude characterized by  $F_e$  external Fermions,  $B_e$  external Bosons or photons and  $V_e$  external vector mesons. Thus, a primitive divergence must satisfy  $K \geq 0$ . What shall be called the 'conventional renormalizability

criterion' arises if a bound for  $K$  is evaluated in the usual manner, that is, if it is assumed that each internally propagated vector meson requires an integration over all momenta, but contributes no compensating factors of momentum from the denominator of its propagator. With the elimination of the parameters counting the numbers of internal lines, this bound is given by

$$K \leq \bar{K} = 4 - \frac{3}{2}F_e - 2V_e - B_e + \sum_i n_i \left( \frac{3}{2}f + 2v + b + s - 4 \right). \quad (2.2)$$

This expression is certain to obscure any fortuitous cancellations of the gradient terms of the vector meson propagators that would reduce the value of  $K$  actually realized. To ensure that primitive divergencies exist for only a finite number of distinct values of  $F_e$ ,  $B_e$  and  $V_e$  (i.e., that the theory is renormalizable), we must demand the inequality

$$n_i \left( \frac{3}{2}f + 2v + b + s - 4 \right) \leq 0 \quad (2.3)$$

for all  $n_i$ , and consequently each term of  $L$  must satisfy the relation

$$C(L_i) = \frac{3}{2}f + 2v + b + s \leq 4. \quad (2.4)$$

Since  $C = 5$  for a 'Yukawa interaction' of the form  $Z_\mu \bar{\psi} \gamma^\mu \psi$  and  $C = 6$  for the 'minimal' electrodynamic interaction contained within  $(\Pi_\mu Z_\nu - \Pi_\nu Z_\mu)^2$ , we see that (2.4) is a sufficiently restrictive condition to exclude all local interactions of physical interest.

However, from (1.8) and (1.9) we notice that the difficulties encountered with the renormalization of vector-meson interactions are associated entirely with the singular propagation of the longitudinal mode. The propagator for the transverse part of  $Z_\mu$  falls off with  $p^{-2}$  for large momenta, behaving in this connection identically to propagators of other boson fields. Consequently, for interactions in which only  $Z_\mu^t$  participates, a sufficient criterion for renormalizability is given by

$$C' = \frac{3}{2}f + t + b + s \leq 4, \quad (2.5)$$

where  $t$  counts the number of factors of  $Z_\mu^t$  appearing in the interaction. The coupling,  $Z_\mu j^\mu$ , of a vector meson to a conserved current differs from  $Z_\mu^t j^\mu$  only by a total derivative. Thus, such a theory is renormalizable provided only that  $C(j_\mu) \leq 3$ , which is easily fulfilled.

Less trivial instances in which the longitudinal mode is also excited require a renormalizability criterion that provides a limitation upon the extent of this excitation. It is the principal observation of this note that  $\bar{K}$  is often an overestimate of the degree of divergence, consequently that (2.4) is an overly severe restriction upon the renormalizable interactions. An improved bound for  $K$  will emerge from a treatment that throughout deals separately with the well-behaved transverse modes and the non-physical longitudinal one.

From the field equations (1.2) follows the supplementary condition that  $Z_\mu$  satisfies:

$$M^2 \partial^\mu Z_\mu = -\partial^\mu j_\mu. \quad (2.6)$$

With the decomposition (1.7) this yields a differential equation for the longitudinal part of the field:

$$M^2 \frac{d^2}{dx_\mu^2} A = -\partial^\mu j_\mu. \quad (2.7)$$

The solution to this equation, generally a non-local function of  $Z_\mu^t$ ,  $Z^{\mu\nu}$ , and the other fields, may be employed to eliminate  $A$  from the interaction Lagrange function to form an effective Lagrange function,  $L'(Z_\mu^t, Z^{\mu\nu}, \dots)$ . Since perturbation theory evaluates transition amplitudes between non-interacting states, these terminal states always satisfy a free-particle supplementary condition,  $\partial^\mu Z_\mu = 0$ . They are thus null eigenvectors of the longitudinal part of  $Z_\mu$ , and the eigenvalues of  $Z_\mu^t$  provide a sufficient specification of their dependence upon the vector meson field. Consequently, the Lagrange function  $L'$  from which  $A$  has been eliminated still describes the system completely.

The modified renormalizability criterion is simply the requirement that the Lagrange function  $L'$ , examined in the usual manner, yields a renormalizable theory. In view of the expressions (1.8) for vector meson propagators involving  $Z_\mu^t$  and  $Z^{\mu\nu}$ , this criterion becomes simply

$$C'(L') \leq 4, \quad (2.8)$$

where each factor of  $Z^{\mu\nu}$  in  $L'$  contributes unity both to  $t$  and to  $s$ . If (2.8) is satisfied there may exist primitively divergent contributions solely to transition amplitudes for processes that satisfy

$$\frac{3}{2}F_0 + V_0 + B_0 \leq 4. \quad (2.9)$$

### 3. Partially-Conserved Currents

It is at this point a useful digression to introduce the definition of a certain limited symmetry property that a Lagrange function may display. Generally  $L$  may be separated into three parts,

$$L = L_K + L_I + L_M, \quad (3.1)$$

where  $L_K$  is the kinematic Lagrange function, the only term to contain explicit differentiations. It is this function that determines both the commutation properties and the expressions for generators of infinitesimal variations of the fields. With the introduction of suitable supplementary fields,  $L_K$  may be written

$$L_K = \frac{1}{2}(\chi \not{\partial}^\mu \partial_\mu \chi - \partial_\mu \chi \not{\partial}^\mu \chi). \quad (3.2)$$

$L_I$  contains the interactions to which the fields are subjected. So that the kinematical properties of the fields do not depend upon coupling strengths, none of these interactions may contain explicit derivatives. It is also attractive to hypothesize that only dimensionless coupling constants are present in  $L_I$ , and consequently that the only admissible interactions are of the forms

$$\begin{array}{ll}
 \text{a. } \psi\psi\phi & \text{d. } Y_{\mu\nu}Y_\mu Y_\nu \\
 \text{b. } Y_\mu\psi\psi & \text{e. } \phi^4 \\
 \text{c. } Y_\mu\phi_\mu\phi & \text{f. } Y_\mu^4 \\
 & \text{g. } Y_\mu^2\phi^2.
 \end{array} \tag{3.3}$$

Here the spinor  $\psi$  represents any spin- $\frac{1}{2}$  field,  $\phi_\mu$  and  $\phi$  represent any spinless field, and  $Y_{\mu\nu}$  and  $Y_\mu$  represent any spin-one field. With the assumption that elementary-particle decays are mediated by a vector meson, these forms are sufficient to describe all interactions known to exist.

$L_M$  contains only the mass producing terms of the theory. It has the form

$$L_M = m\bar{\psi}O_1\psi + \mu^2\phi O_2\phi + M^2Y_\mu O_3Y_\mu, \tag{3.4}$$

in which the  $O_i$  are dimensionless matrices.

If the fields are subjected to an infinitesimal unitary transformation,  $\chi \rightarrow (1 + \mathcal{F})\chi$ , three sorts of invariance properties of the Lagrange function may be distinguished:

1) A transformation under which  $L_K$  is invariant also leaves the commutation relations unchanged and is consequently a canonical transformation. The condition for  $\mathcal{F}$  to be canonical is given by  $[\mathcal{A}_\mu, \mathcal{F}] = 0$ .

2) If  $\mathcal{F}$  is a symmetry of the entire Lagrange function, there must correspond to it the conserved current,  $J_\mu(\mathcal{F})$ . For example, phase transformation of charged fields is an invariance resulting in the continuity equation for the electromagnetic current.

3) If  $L_K$  and  $L_I$ , but not  $L_M$ , are invariant then  $\mathcal{F}$  will be called a 'partial symmetry' of the Lagrangian. The current corresponding to  $\mathcal{F}$  is not conserved, but satisfies what shall be called a 'partial conservation law' of the form

$$\partial^\mu j_\mu = m\bar{\psi}P_1\psi + \mu^2\phi P_2\phi + M^2Y_\mu P_3Y_\mu \tag{3.5}$$

where the  $P_i$  are dimensionless numerical matrices given by

$$P_1 = O_1\mathcal{F} - \beta\mathcal{F}\beta O_1, \quad P_{2,3} = [O_{2,3}, \mathcal{F}]. \tag{3.6}$$

If the masses are set equal to zero,  $\mathcal{F}$  becomes an invariance of the entire Lagrangian, and (3.5) a conservation law.

The relation of this discussion to that of section 2 is perhaps less obscure with the observation that in general  $C'(\partial^\mu j_\mu) = C'(j_\mu) + 1$ , while for a partially-conserved current  $C'(\partial^\mu j_\mu) = C'(j_\mu)$ .

### 4. Some Renormalizable Interactions

An interaction of the sort  $m_{\mu\nu} Z^{\mu\nu}$ , where  $m_{\mu\nu}$  does not itself depend upon the vector meson fields, is renormalizable only if  $C(m_{\mu\nu}) \leq 2$ . There is no such antisymmetric tensor that may be constructed of spin- $\frac{1}{2}$  and spin-0 fields and their derivatives, and hence no such renormalizable local interaction.

To investigate the renormalizability of a vector interaction,  $j^\mu Z_\mu$ , we must first evaluate and eliminate the longitudinal part of the vector field. For such currents that  $\partial^\mu j_\mu$  does not explicitly depend upon  $Z_\mu$ , (2.7) yields the expression

$$A(x) = M^{-2}(x|p^{-2}|x')\partial^\mu j_\mu(x'), \tag{4.1}$$

and the effective interaction Lagrange function becomes

$$j^\mu Z_\mu - \partial^\mu j_\mu(x)(x|p^{-2}|x')\partial^\nu j_\nu(x'). \tag{4.2}$$

The resulting theory is renormalizable if both the inequalities

$$C'(j_\mu) \leq 3 \quad \text{and} \quad C'(\partial^\mu j_\mu) \leq 3 \tag{4.3}$$

are satisfied. The first restriction requires a local  $j_\mu$  to be of the form  $\bar{\psi} O_\mu \psi$  or  $\phi_\mu O\phi$ . The second inequality limits  $\partial^\mu j_\mu$  to the general form

$$\partial^\mu j_\mu = \bar{\psi}(O + O'\gamma_5)\psi + \mathcal{P}_3(\phi, \dot{\phi}), \tag{4.4}$$

where  $O$  and  $O'$  are dimensionless matrices and  $\mathcal{P}_3$  is a polynomial in  $\phi$  and its derivatives of at most third degree. Note that (4.4) includes any 'partially-conserved current' which by definition must satisfy (3.5). So long as the first inequality of (4.3) is met, any interaction of a vector meson with a partially-conserved current will be renormalizable.

An example of a theory admitted as renormalizable by the new criterion, but excluded by the conventional one, is described by the Lagrange function

$$L = L_K(\psi, Z) + m\bar{\psi}\psi + M^2 Z_\mu Z^\mu + gZ_\mu \bar{\psi}\gamma^\mu (i\nu + ia\gamma_5)\psi, \tag{4.5}$$

in which  $Z_\mu$  is a neutral vector field and  $\psi$  is a real Fermi field with a single internal degree of freedom to which  $\nu = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$  refers. The interaction current satisfies the partial-conservation law

$$\partial^\mu [\psi\gamma_\mu (\nu + a\gamma_5)\psi] = -2iam\bar{\psi}\gamma_5\psi. \tag{4.6}$$

The analogous coupling of a charged vector meson is not, by itself, renormalizable. For example, with the interaction

$$Z_\mu^k (\bar{\psi}\gamma^\mu i t_k \psi) \quad (k = 1, 2) \tag{4.7}$$

of a charged vector meson to a  $T = 1$  triplet of Fermions, the current now satisfies

$$\partial^\mu (\bar{\psi}\gamma_\mu t_k \psi) = e^{kj3} Z_\mu^j (\bar{\psi}\gamma^\mu t^3 \psi)$$

which is not of the form (4.4), and this interaction does not qualify as

renormalizable. Thus, a renormalizable theory of the beta-interactions cannot be generated by the interactions of a charged vector meson without including appropriate self-couplings. We would like to be able to identify these self-couplings with the electrodynamic interactions of the vector meson.

### 5. Electrodynamic Interactions of Vector Mesons

We shall represent the charged vector meson by two real fields, with the charge given by the  $2 \times 2$  antisymmetric imaginary matrix  $t_3$ . The most general linear electrodynamic coupling not involving explicit differentiations is given by

$$L_I = ie[Z^{\mu\nu}t_3(Z_\mu A_\nu - Z_\nu A_\mu) + \gamma F^{\mu\nu}Z_\mu t_3 Z_\nu], \quad (5.1)$$

in which the first term corresponds to the usual 'minimal' interaction and an additional 'anomalous magnetic moment' interaction has been introduced. The consequent field equation for  $Z_\mu$ , where  $\Pi_\mu$  denotes  $\partial_\mu + it_3 A_\mu$ , is  $(\Pi^2 - M^2)Z_\nu$

$$= -iet_3[(1+\gamma)F_{\nu\mu}Z^\mu + \Pi_\nu \frac{1}{2}M^{-2}((1-\gamma)F_{\lambda\sigma}Z^{\lambda\sigma} - 2\gamma j_\mu Z^\mu)]. \quad (5.2)$$

In the non-relativistic limit, and with the neglect of the last term which represents a direct self-interaction, (5.2) becomes

$$[(\frac{1}{2}M)\Pi^2 - eA_0]Z + (e/2M)(1+\gamma)(Z \times \mathcal{E}) + (e/4M^2)(1-\gamma)\Pi(Z \cdot \mathcal{E}) = 0. \quad (5.3)$$

Evidently  $Z$  possesses a magnetic dipole moment of size  $(e/2m)(1+\gamma)$  and an electric quadrupole moment of size  $(e/4m^2)(1-\gamma)$ , in units wherein  $\hbar = c = 1$ .

In order to investigate the renormalizability of this theory, we must express the longitudinal part of  $Z_\mu$  in terms of the other fields, and eliminate it from the Lagrange function. Writing the interaction as

$$L_I = ie(2Z^{\mu\nu}t_3 Z_\mu^i A_\nu + \gamma F^{\mu\nu}Z_\mu^i t_3 Z_\nu^i) + L_A, \quad (5.4)$$

where  $L_A$  is the part of  $L_I$  involving  $A$ ,

$$L_A = ie(2Z^{\mu\nu}A_\nu - \gamma F^{\mu\nu}Z_\nu^i - \gamma F^{\mu\nu}\partial_\nu A)t_3 \partial_\mu A, \quad (5.5)$$

we observe immediately that  $L_I - L_A$  satisfies the renormalization criterion (2.8). Thus the renormalizability depends entirely upon the behaviour of the longitudinal mode; it is renormalizable if  $C'(L_A) \leq 4$  or equivalently, if  $C'(A) \leq 0$ .

We determine  $A$  from the supplementary condition  $Z_\mu$  must satisfy. From (5.1) follows

$$M^2 \partial^\mu Z_\mu = iet_3 \partial_\nu [A_\mu Z^{\mu\nu} - \gamma F^{\mu\nu} Z_\mu], \quad (5.6)$$

which may be reduced with the use of the field equations to

$$M^2 \partial^\mu Z_\mu = iet_3 [\frac{1}{2}(\gamma-1)F^{\mu\nu}Z_{\mu\nu} - M^2 A^\mu Z_\mu]. \quad (5.7)$$



Therefore  $\Lambda$  satisfies the differential equation

$$(d^2/dx_\mu^2 + iet_3 A^\mu d/dx_\mu) \Lambda = iet_3 (A^\mu Z_\mu^\dagger + \frac{1}{2}(1-\gamma)M^{-2} F^{\mu\nu} Z_{\mu\nu}). \quad (5.8)$$

Defining an integral operator

$$(x|\Omega|x') = \int (2\pi)^{-2} d^4 p e^{ip(x-x')} i t_3 A_\mu(x) p^{-2} \hat{p}^\mu,$$

we may write the solution to (5.8) as a power series:

$$\Lambda(x) = \left( \sum_{n=0}^{\infty} e^n \Omega^n \right) p^{-2} iet_3 (A^\mu Z_\mu^\dagger + \frac{1}{2}M^{-2}(1-\gamma)F^{\mu\nu}Z_{\mu\nu}). \quad (5.9)$$

For each term of this series, from (2.5), we have  $C' = 2$ , and consequently  $C'(L_\Lambda) = 6$ . In general the renormalization criterion is not met. However, if  $\gamma$  is chosen as unity so that the second bracketed term of (5.9) vanishes, we obtain  $C' = 0$  for each term. In this case, inserting (5.9) into (5.4),  $L'$  is expressed as a power series in  $e$  each of whose partial sums is characterized by  $C' = 4$ . From (2.9) it follows that primitive divergences exist only for transition amplitudes with  $V_e \leq 4$ . Because  $L'$  is an infinite sum, there will be an infinite number of distinct primitive divergences contributing to each such amplitude. Nevertheless, the introduction of a finite number of counter-terms yields a finite theory, and the electrodynamics of a vector meson with unit anomalous moment is thus shown to be renormalizable. We must emphasize that this result applies only to the isolated vector meson. If other charged fields are present, an additional term will appear in (5.7), which destroys the renormalizability.

### 6. Discussion

It should be observed that the choice  $\gamma = 1$ , besides leading to a renormalizable electrodynamics, is remarkable for certain apparently unrelated reasons. It assures that the uncorrected gyromagnetic ratio of  $Z_\mu$  is two, as well as that  $Z_\mu$  displays no electric quadrupole moment. Furthermore, if the  $Z$ -meson mass is to be regarded as a consequence of some unspecified interaction, then we must be able to examine the theory in the absence of this interaction. In this limit, (5.7) becomes

$$\frac{1}{2}(1-\gamma)t_3 F^{\mu\nu} Z_{\mu\nu} = 0.$$

Unless  $\gamma = 1$ , this relation conflicts with the assumed kinematical independence of the vector meson and the electrodynamic fields, hence it is only the renormalizable theory for which there exists a consistent zero-mass limit<sup>4</sup>).

In deriving (5.7) from (5.6) we used the relation

$$\partial_\nu (A_\mu Z^{\mu\nu} - \gamma Z_\mu F^{\mu\nu}) = \frac{1}{2}(\gamma-1)F^{\mu\nu}Z_{\mu\nu} - M^2 A^\mu Z_\mu. \quad (6.1)$$

Thus, with the choice  $\gamma = 1$ , the vector  $A_\mu Z^{\mu\nu} - \gamma Z_\mu F^{\mu\nu}$  becomes what we have defined by (3.5) to be a partially conserved current. It is illuminating to

construct the infinitesimal transformation which displays the corresponding 'partial symmetry' of the Lagrange function. Defining  $Z_\mu^3 = A_\mu$  and  $Z_{\mu\nu}^3 = F_{\mu\nu}$  we may re-express (5.1) with  $\gamma = 1$  as

$$L_I = e e^{ijk} Z_i^{\mu\nu} Z_\mu^j Z_\nu^k, \quad (6.2)$$

which is formally identical to the vector-meson interaction discussed by Yang and Mills <sup>5)</sup> in another context. As they observe, (6.2) is invariant under 'I-spin' rotations of the Z-meson triplet,

$$\begin{aligned} Z_\mu &\rightarrow (1 + it \cdot \theta) Z_\mu \\ Z_{\mu\nu} &\rightarrow (1 + it \cdot \theta) Z_{\mu\nu}. \end{aligned} \quad (6.3)$$

In our case, the identification of  $Z^3$  with light prevents a complete symmetry of this sort. The mass producing part of  $L$  cannot produce a photon mass, hence we must have

$$L_M = -\frac{1}{2} M^2 Z_\mu^3 t_3^2 Z_\mu. \quad (6.4)$$

Consequently only the  $t_3$  rotation is an invariance of the entire Lagrange function; the other rotations are now only partial-symmetries. Of the corresponding currents,

$$j_\mu^i = e^{ijk} Z_{\mu\nu}^j Z_\nu^k, \quad (6.5)$$

only  $j_\mu^3$ , the electrical current, is conserved. The others satisfy partial-conservation laws. More succinctly, we have

$$\partial^\mu j_\mu = -M^2 A^\mu t_3 Z_\mu, \quad (6.6)$$

which reproduces (6.1) with  $\gamma = 1$ .

If other fields are introduced which also bear charge, the partial conservation of  $j_\mu^3$  is destroyed. Thus, if we include nucleons, the Lagrange function

$$L_e = e \left( \sum_{i,j,k=1}^3 e^{ijk} Z_i^{\mu\nu} Z_\mu^j Z_\nu^k + Z_\mu^3 i\bar{\psi} \gamma^\mu \frac{1}{2} (\tau_3 + \zeta_3) \psi \right) \quad (6.7)$$

possesses only the  $|t_3|$  invariance, and does not generate a renormalizable theory. Similarly, the prototype beta-interaction

$$L_\beta = \beta \sum_{i=1,2} Z_\mu^i \bar{\psi} O_i \gamma^\mu \psi \quad (6.8)$$

(in which the  $O_i$  may refer to charge, parity, strangeness, etc.) is not renormalizable in view of the discussion near (4.7). However, it is interesting to note that by combining these two interactions we may secure a renormalizable theory. Recalling that

$$\frac{\beta^2}{M^2} \approx \frac{10^{-5}}{m_N^2},$$

we may identify  $\beta$  with  $e$  providing that the vector meson mass is taken to be about thirty nucleon masses. The combined interactions may then be written

$$L_W = e \left( \sum_{i,j,k=1}^3 \varepsilon^{ijk} Z_i^{\mu\nu} Z_\mu^j Z_\nu^k + \sum_{i=1}^3 Z_\mu^i \bar{\psi} \gamma^\mu O_i \psi \right), \quad (6.9)$$

where  $O_3$  is defined as to be  $\frac{1}{2}i(\tau_3 + \zeta_3)$ . This theory is renormalizable if the  $O_i$  constitute a representation of the rotation group,

$$[O_i, O_j] = i\varepsilon^{ijk} O_k, \quad (6.10)$$

for then the infinitesimal transformation,  $\psi \rightarrow (1 + \mathbf{O} \cdot \boldsymbol{\theta})\psi$ , together with (6.3) constitutes a partial-symmetry of  $L_W$ .

From the corresponding partial-conservation law follows

$$\partial_\mu^2 A = -A^\mu t_3 Z_\mu,$$

and the arguments of section 5 may again be employed to show that the renormalization criterion is met.

To obtain a realistic theory of the decay processes, we must show that  $O_{1,2}$  may contain both symmetric parity violation and the known strangeness selection rules. We must also include the existence of mesons and their strong couplings. The discussion of these points is reserved for a more speculative sequel.

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