

Chapter 4

Probing the Internal Structure of the Proton

The protons and neutrons are the basic building blocks of atomic nuclei. The internal structure of the nucleons determines their fundamental properties, which in turn directly affect the properties of the nuclei. Understanding how the nucleon is built in terms of underlying quark and gluon degrees of freedom is one of most important and challenging questions in modern nuclear physics. In the last chapter, we discussed a simple quark model for the structure of the nucleon. This model, however, is rather crude and must be tested against more extensive and accurate experimental data. Eventually, we hope to get a picture of the nucleon based on QCD theory.

The structure of the nucleon can be studied by scattering electrons (and also muons), in a similar way that the structure of condensed matter or atoms can be studied through scattering of X-rays, neutrons, and electrons. There are two basic reasons why the electromagnetic interaction is a good tool for taking a “picture” of the nucleon:

- QED is a “known” interaction
- $\alpha_{em} = \frac{1}{137} \Rightarrow$ perturbation theory is valid.

So we have a well-defined interaction for our probe and a systematic calculational scheme for computing the results of experiments.

Electrons, being common charged particles, can easily be accelerated in well-defined monoenergetic beams and accurately detected using magnetic spectrometers and standard particle detection techniques. This technique offers superior control over the kinematic properties of the exchanged virtual photon that probes the hadronic system.

There are two types of scattering which are most useful in studying the nucleon. First is elastic scattering in which the final state of the nucleon remains the same, but with a finite recoil. In this case, the scattering cross section allows one to map out the charge and density distributions inside the nucleon. The second type is the deep-inelastic scattering (DIS) in which a quark in a nucleon gets knocked out by the virtual photon, and the proton gets smashed into many fragments. This process allows one to extract the quark and gluon distributions in momentum space or Feynman parton distributions. In the past few decades, both processes have taught us a great deal about the structure of the nucleon and are still in use in laboratories around the world.

4.1 Elastic Electron-Nucleon Scattering

Elastic e^- -N scattering is depicted in terms of a Feynman diagram in Fig. 4.1, where the dominant one-photon exchange interaction is shown. The incident and outgoing electrons have 4-momenta $k = (E, \vec{k})$ and $k' = (E', \vec{k}')$, and the initial and final nucleon 4-momenta are P and P' , respectively. We are interested in the electrons with energy much larger than their rest mass. The 4-momentum of the virtual photon is

$$q = k - k' = P' - P . \quad (4.1)$$

The invariant mass of the photon is $q^2 = -4EE' \sin^2 \theta/2$, which is always negative (or space-like by definition). This means that we can always find a frame in which the energy transfer $q^0 \equiv \nu = 0$ and $q^2 = -\vec{q}^2 \equiv -Q^2$. [The so-called Breit frame is such a frame]. The elastic scattering condition in the lab frame gives,

$$(P')^2 = (P + q)^2 = M^2 \quad (4.2)$$

$$= M^2 + 2P \cdot q + q^2 \quad (4.3)$$

$$= M^2 + 2M\nu + q^2 \quad (4.4)$$

Therefore, $2M\nu = Q^2$.

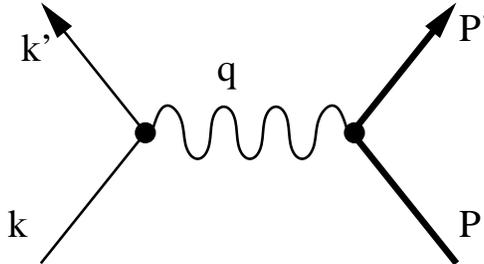


Figure 4.1: Elastic scattering between charged lepton and proton through one photon exchange interaction.

Experimentally, one arranges a mono-energetic beam of electrons incident on a nucleon target, and detects the outgoing electrons (spectrum of energies E') at some scattered angle θ . Thus, the incident and scattered momenta of the electrons fix all components of the 4-vector q . Since for a given θ there is only one allowed energy loss $\nu = E - E'$, the spectrum of ν is just a delta function at $\nu = -q^2/2M$ corresponding to the scattered energy $E' = E/(1 + \frac{2E}{M} \sin^2 \frac{\theta}{2})$.

The physical observables characterizing compositeness are form factors, which enter the elastic scattering cross section. In condensed matter physics, it is also the form factors (or structure factors) that are probed X-ray or electron scattering which, roughly speaking, are the Fourier transformation of some density distributions. In atomic physics, the form factor of the hydrogen atom is the Fourier transformation of the charge density. In fact, many 3-dimensional image reconstructions are done from the inverse Fourier transformation of a certain scattering cross section.

Let us consider the scattering cross section in one-photon exchange, and seek connection to the proton compositeness. If the electron-photon vertex is $-ie\gamma^\mu$, the electron proton vertex is

$ie\langle P'|J^\mu|P\rangle$. The S -matrix element then reads

$$\begin{aligned} S &= (2\pi)^4 \delta^4(k + P - P' - k') \bar{u}(k') (-ie\gamma^\mu) u(k) \frac{-i}{q^2} \langle P'| (ie) J^\mu |P\rangle \\ &= -i(2\pi)^4 \delta^4(k + P - P' - k') \mathcal{M} . \end{aligned} \quad (4.5)$$

where \mathcal{M} is called invariant amplitude. The electromagnetic current is,

$$J^\mu(\xi) = \sum_i e_i \bar{\psi}_i(\xi) \gamma^\mu \psi_i(\xi) \quad (4.6)$$

where i sums over all quark flavors: up, down, strange, charm, beauty, and top. The first three quarks are light compared with the mass of the nucleon and are mostly what we will consider in this course. The heavier quarks are ignored because they are heavy and play relatively minor role.

In terms of the invariant amplitude, the elastic scattering cross section reads

$$d\sigma = \frac{1}{2k^0 2P^0 |v_1 - v|} (2\pi)^4 \delta^4(k + P - P' - k') |\mathcal{M}_{fi}|^2 \prod_f \frac{d^3 p_f}{2E_f (2\pi)^3} , \quad (4.7)$$

where v_1 is the electron velocity, v is the initial nucleon velocity, and $2k^0 2P^0 |v_1 - v|$ is invariant when boosted along the z -direction. We can write it in the following invariant form $4I = 4\sqrt{(k \cdot P)^2 - m^2 M^2}$. The last factor is the phase space whose form is closely related to the normalization we choose. In the laboratory frame, $k = E$, $P^0 = M$, and we can integrate over the proton momentum by writing the phase factor as $2\pi \delta(P'^2 - M^2) d^4 P' / (2\pi)^4$ to obtain

$$d\sigma = \frac{1}{2ME} |\mathcal{M}|^2 2\pi \delta((q + P)^2 - M^2) \frac{d^3 \vec{k}'}{2E' (2\pi)^3} . \quad (4.8)$$

Integrating over $E' = |\vec{k}'|$, we obtain

$$d\sigma = \frac{E'}{2EM^2} \frac{1}{1 + \frac{2E}{M} \sin^2 \frac{\theta}{2}} |\mathcal{M}|^2 \frac{d\Omega}{(2\pi)^2} . \quad (4.9)$$

where the recoil factor is

$$f_{\text{rec}} = \frac{1}{1 + \frac{2E}{M} \sin^2 \frac{\theta}{2}} , \quad (4.10)$$

which reduces to unity if the particle is infinitely heavy.

The invariant amplitude squared is

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} \ell^{\mu\nu} \langle P|J^\nu|P'\rangle \langle P'|J^\mu|P\rangle , \quad (4.11)$$

where $\ell^{\mu\nu}$ is the lepton tensor,

$$\ell^{\mu\nu} = \bar{u}(k') \gamma^\mu u(k) \bar{u}(k) \gamma^\nu u(k') \quad (4.12)$$

For unpolarized scattering, we average over the initial polarization and sum over the final polarization states to obtain

$$\ell^{\mu\nu} = 2(k'^\mu k^\nu + k'^\nu k^\mu - g^{\mu\nu} k' \cdot k) . \quad (4.13)$$

The hadron tensor $W^{\mu\nu} = \langle P|J^\nu|P'\rangle\langle P'|J^\mu|P\rangle$ depends on the current matrix element on which we now focus.

The matrix element of the current between the nucleon states defines two form factors,

$$\langle P'|J^\mu(0)|P\rangle = \bar{U}(P') \left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M} \right] U(P) \quad (4.14)$$

$F_1(Q^2)$ is called the Dirac form factor and $F_2(Q^2)$ is the Pauli form factor. As we shall see later, it is physically more interesting to introduce the so-called Sachs electric and magnetic form factors,

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \tau F_2(Q^2) \\ F_2(Q^2) &= F_1(Q^2) + F_2(Q^2) . \end{aligned} \quad (4.15)$$

where $\tau = Q^2/4M^2$.

The hadron tensor now becomes

$$\begin{aligned} W^{\mu\nu} &= 2(P'^\mu P^\nu + P'^\nu P^\mu - g^{\mu\nu}(PP' - M^2))G_M^2 \\ &\quad - 2F_2G_M(P + P')^\mu(P + P')^\nu \\ &\quad + F_2^2\frac{M^2 + P \cdot P'}{2M^2}(P + P')^\mu(P + P')^\nu \\ &= (-q^\mu q^\nu + g^{\mu\nu}q^2)G_M^2 + (P + P')^\mu(P + P')^\nu\frac{G_E^2 + \tau G_M^2}{1 + \tau} \\ &= g^{\mu\nu}q^2G_M^2 + 4P^\mu P^\nu\frac{G_E^2 + \tau G_M^2}{1 + \tau} + \dots \end{aligned} \quad (4.16)$$

where the ellipses denote terms involving factors of q^μ which does not contribute to the cross section because of the current conservation.

Contract the lepton and hadron tensors, and recall that the both tensors are symmetric and conserved, $q^\mu\ell_{\mu\nu} = q^\mu W_{\mu\nu} = 0$. In the lab frame, the elastic scattering cross section becomes

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right] , \quad (4.17)$$

where the Mott scattering cross section represents the scattering of the electron from a point-like scalar proton

$$\sigma_{\text{Mott}} = \frac{Z^2\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} f_{\text{rec}} \quad (4.18)$$

If the proton were structureless, then $G_E = G_M = 1$. We obtain a cross section

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \left[1 + 2\tau \tan^2 \frac{\theta}{2} \right] . \quad (4.19)$$

Any observed deviation from this is a clear indication of nucleon substructure.

The cross section can be rewritten as

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{Mott}}}{1 + \tau} \left[G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \right] , \quad (4.20)$$

where $\epsilon^{-1} = 1 + (1 + \tau)2 \tan \theta/2$ is the virtual photon longitudinal polarization. To extract separately the electric and magnetic form factors from elastic scattering, one must measure two cross sections at fixed Q^2 by varying the scattering angle and hence ϵ . When plotting the quantity in the square bracket versus the quantity ϵ^{-1} , the intercept and slope provide separately the electric and magnetic form factors. This is called Rosenbluth separation method. However, at high- Q^2 , the magnetic form factor contribution is much larger, the electric form factor is difficult to extract with this method. We will come to this point later.

4.2 Physics of the Form Factors and Experimental Data

If the nucleon is very heavy, $M \rightarrow \infty$, the momentum and position eigen-states are the same. The initial and final state nucleons are fixed at the same location, never moved during the scattering, although the external photon brings in some momentum change, $Q^2 \ll M^2$. Thus, we may consider the initial and final states to consist of the same internal state. In this case, the interpretation of form factors is just like the case of condensed matter or atomic physics, and the Fourier transformation of the form factors is just the density distributions.

However, the nucleon mass is finite, and the nucleon recoil effect becomes important in its interpretation. The initial and final state nucleon wave functions are not sampled in the same frame and hence there is a relative Lorentz contraction. There is no known model-independent way to separate the internal structure effects from the recoil kinematical effects, although many model approaches have been suggested in the literature. To minimize this effect, one may consider the scattering in the Breit frame in which the initial and final state nucleons have momenta with the same magnitude, hence similar Lorentz contraction effect. Let us consider the virtual photon absorption cross sections in this special frame.

Given the photon momentum q^μ , the longitudinal polarization vector (time-like) is

$$\epsilon_L^\mu = (q^3, 0, 0, q^0)/Q, \quad (4.21)$$

which satisfies the condition that $q \cdot \epsilon = 0$. [This condition is necessary in the Lorentz gauge where $\partial_\mu A^\mu = 0$.] In the Breit frame, the polarization vector becomes $\epsilon_L^\mu = (1, 0, 0, 0)$. We find then the charge density matrix element

$$\begin{aligned} \epsilon_L^\mu \langle P' | J_\mu | P \rangle &= \langle P' | J^0 | P \rangle \\ &= \bar{U}(P') \left[[F_1(Q^2) + F_2(Q^2)] \gamma^0 - \frac{E_P}{M} F_2(Q^2) \right] U(P). \end{aligned} \quad (4.22)$$

It is easy to show that $\bar{U}(P') \gamma^0 U(P) = 2M$ and $\bar{U}(P') U(P) = 2E_P$. Thus,

$$\begin{aligned} \epsilon_L^\mu \langle P' | J_\mu | P \rangle &= 2M \left[F_1(Q^2) - \frac{P^2}{M^2} F_2(Q^2) \right] \\ &= 2M \left[F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2) \right] \\ &= 2MG_E(Q^2). \end{aligned} \quad (4.23)$$

so we arrive at the Sachs electric form factor as the matrix element of the electric charge density in the Breit frame ($q^0 = 0$). Then $G_E(Q^2)$ may be interpreted as the Fourier transformation of the

charge distribution. If we expand it at small Q^2 ,

$$\begin{aligned} \int e^{i\vec{q}\vec{r}} \rho(r) d^3r &= \int (1 - i\vec{q}\vec{r} - \frac{1}{2}(\vec{q}\vec{r})^2 + \dots) \rho(r) d^3r \\ &= Q_e - \frac{1}{6} q^2 \int r^2 \rho(r) d^3r + \dots \\ &= Q_e - \frac{1}{6} q^2 \langle r^2 \rangle + \dots \end{aligned} \quad (4.24)$$

Then

$$G_E(Q^2) = Q_e - \frac{1}{6} Q^2 \langle r^2 \rangle + \dots \quad (4.25)$$

Therefore, we define the charge radius of the nucleon as

$$\langle r^2 \rangle = -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0} . \quad (4.26)$$

Other definitions of charge radius are certainly possible, but this is the most common one.

The transverse polarization vector is given by

$$\epsilon_T^\mu(\lambda = \pm 1) = \mp(0, 1, \pm i, 0)/\sqrt{2} . \quad (4.27)$$

Therefore we have

$$\begin{aligned} \epsilon_T^\mu(\sigma = 1) \langle P' s_z = 1/2 | J_\mu | P s_z = -1/2 \rangle &= \bar{U}(P') \gamma^\mu \epsilon_\mu(\sigma = 1) U(P) [F_1(Q^2) + F_2(Q^2)] \\ &= -Q/\sqrt{2} G_M(Q^2) . \end{aligned} \quad (4.28)$$

where $G_M(Q^2) = F_1 + F_2$ is the Sachs magnetic form factor. The polarization vector selects $J_x + iJ_y$ components of the current. Since it is the helicity amplitudes which appear directly in the cross section, the magnetic form factor is always suppressed and enhanced with a factor of Q , at small and large Q . According to the above, $QG_M(Q^2)$ is the Fourier transformation of the transverse current distribution in a polarized proton in the Breit frame.

One can calculate the magnetic moment from the form factors without the effect of the recoil. The definition of the magnetic moment is

$$\mu(2\vec{S}) = \langle PS | \frac{1}{2} \int d^3\vec{r} \vec{r} \times \vec{j} | PS \rangle . \quad (4.29)$$

A somewhat lengthy calculation using the form factor equation yields that,

$$\mu = F_1(0) + F_2(0) . \quad (4.30)$$

in the basic unit of $e\hbar/2mc$. Since $F_1(0) = 1$ for proton and 0 for the neutron, $F_2(0)$ is called the anomalous magnetic moment of the nucleon.

The proton electric and magnetic form factors were first measured at former SLAC in the mid 1950's by R. Hofstadter and collaborators, who won Nobel prize in 1961 for "discovering the internal structure in the protons and neutrons". In these experiments, electrons with energies of several hundred MeV were scattered on nucleon targets. By comparing the scattering cross sections with that of the point-like proton, Hofstadter found that the nucleon is a diffusely extended object.

The proton's electric form factor has been measured now to $Q^2 \sim 5 \text{ GeV}^2$. Because of its small contribution to the cross section at large Q^2 , the Rosenbluth separation yielded controversial results. More recently, the recoil polarization technique has been employed to obtain more reliable measurements of G_E^p . When the scattering electron is longitudinally polarized, the recoil proton has polarization in the scattering plane. The polarizations transverse and parallel to the momentum of the nucleon are given by

$$\begin{aligned} I_0 P_t &= -2\sqrt{\tau(1+\tau)}G_E G_M \tan \theta/2 \\ I_0 P_l &= \frac{E+E'}{M}\sqrt{\tau(1+\tau)}G_M^2 \tan^2 \theta/2 \end{aligned} \quad (4.31)$$

where $I_0 = G_E^2 + \tau/\epsilon G_M^2$. By forming the ratio of the polarizations, one obtains

$$\frac{G_E}{G_M} = -\frac{P_t}{P_l} \frac{E+E'}{2M} \tan \frac{\theta}{2} \quad (4.32)$$

which can be used to extract G_E .

The proton magnetic form factor has been measured up to $Q^2=30 \text{ GeV}^2$. The magnetic moment of the proton was first measured by O. Stern in 1930's. He was awarded Nobel prize for the measurement. The latest number is $\mu_P = 2.792847337(29)\mu_N$.

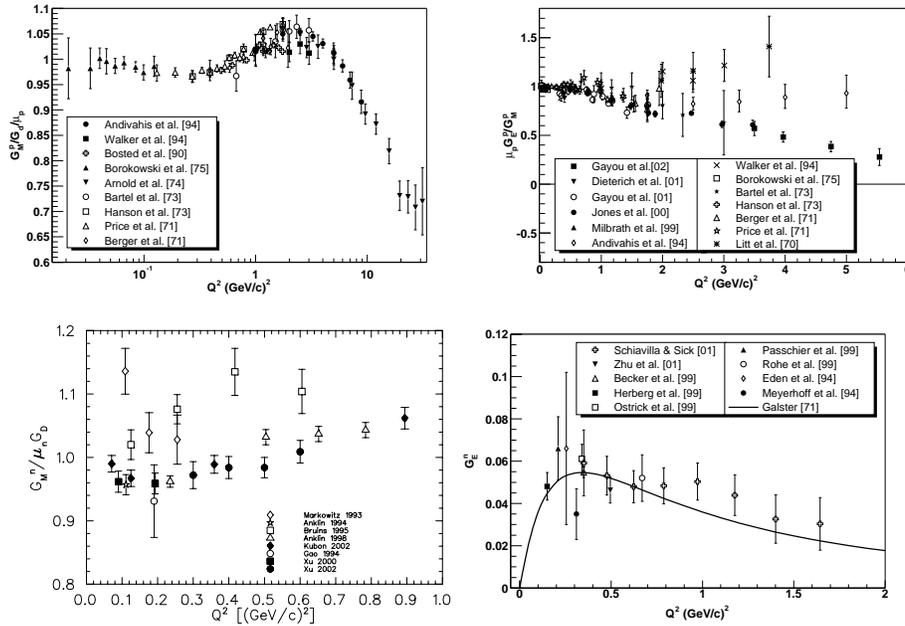


Figure 4.2: Experimental data on the electric and magnetic form factors of the nucleon.

Since there is no free neutron target, one must use either deuteron or ^3He as a target, selecting the so-called quasi-elastic scattering kinematic in the sense that it is similar to just a scattering off a moving neutron. The best measurements are obtained using polarization observables to separate the electric and magnetic form factors. One can either use a polarized d or ^3He target, scatter a

polarized electron beam, and observe the so-called double spin asymmetry. Another method is to scatter a polarized electron and measure the neutron polarization. G_E^n measurements have been carried out up to about 1.5 GeV^2 . By scattering thermal neutrons from the stationary electrons in solids, one finds the neutron charge radius is

$$\langle r_c^2 \rangle_n = -0.116 \text{ fm}^2 . \quad (4.33)$$

The Foldy contribution to the charge radius $3\kappa/2M^2$ is almost entirely the contribution of the charge radius. This might be just some numerical coincidence.

The neutron magnetic form factor has been measured up to Q^2 of 10 GeV^2 , using deuteron targets. More precise measurements using polarized beams and targets have been performed recently, and more polarization experiments at higher Q^2 are planned.

A recent compilation of experimental data for the electric and magnetic form factors of the nucleon are shown in Figure 4.2. At low $Q^2 < 1 \text{ GeV}^2$, useful approximate expressions for G_E^p , G_M^p , and G_M^n are given by a phenomenological parametrization known as the ‘‘dipole’’ form factors:

$$G_E^p(Q^2) = G_M^p(Q^2)/\mu_p \quad (4.34)$$

$$= G_M^n(Q^2)/\mu_n \quad (4.35)$$

$$= \frac{1}{(1 + Q^2/Q_0^2)^2} \equiv G_D \quad (4.36)$$

where $Q_0^2 = 0.71 \text{ GeV}^2$. The neutron electric form factor G_E^n is well described by the so-called ‘‘Galster’’ formula:

$$G_E^n(Q^2) = \frac{1.91\tau}{1 + 5.6\tau} G_D . \quad (4.37)$$

These approximations are useful if one just needs to get a qualitative picture.

4.3 Strangeness and Electroweak Form Factors

Although the nucleons have no net strangeness, the nucleon does have a ‘‘sea’’ of $q\bar{q}$ pairs. The charm quarks and heavier quarks are not expected to be present in appreciable amount due to the large masses that need to be created. However, the light sea quarks (including the strange quarks) will be present and represent an interesting aspect of nucleon structure beyond the simple quark models. The sea of gluons and $q\bar{q}$ pairs are present as internal dynamical degrees of freedom in the valence quarks used in the quark models, and are responsible for the heavy mass of the objects considered in the quark models. The $\bar{s}s$ pairs in the nucleon can contribute to the electromagnetic form factors, and this contribution can be studied by measuring neutral weak form factors and combining the results with the well-known electromagnetic form factors.

If we use ψ to represent a column of u,d,s quarks, then we can define the octet vector current,

$$V_a^\mu = \bar{\psi} \gamma^\mu \frac{\lambda^a}{2} \psi . \quad (4.38)$$

If quark masses are not zero, only the diagonal current is conserved since the divergences of the nondiagonal currents are proportional to the quark masses. When the quark masses are small,

we can approximately consider them as conserved. The electromagnetic current is then just the following combination

$$J_{EM}^\mu = V_3^\mu + \frac{2}{\sqrt{3}}V_8^\mu . \quad (4.39)$$

where $V_3^\mu = 1/2(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)$ and $V_8^\mu = (1/2\sqrt{3})(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d - 2\bar{s}\gamma^\mu s)$. In terms of isospin group SU(2), the V_3^μ is a isovector current and V_8^μ is an isoscalar electromagnetic current. Because of the isospin symmetry, the isoscalar current has the same matrix element between the neutron and proton state, where as the isovector current has matrix elements with opposite signs. Therefore, defining the isoscalar and isovector electromagnetic form factors, we have

$$\begin{aligned} G^p(Q^2) &= G^{T=1}(Q^2) + G^{T=0}(Q^2) , \\ G^n(Q^2) &= -G^{T=1}(Q^2) + G^{T=0}(Q^2) . \end{aligned} \quad (4.40)$$

Or inversely,

$$\begin{aligned} G^{T=1}(Q^2) &= (G^p(Q^2) - G^n(Q^2))/2 , \\ G^{T=0}(Q^2) &= (G^p(Q^2) + G^n(Q^2))/2 . \end{aligned} \quad (4.41)$$

The matrix elements of the proton and neutron allow one to extract the matrix elements of the isovector and isoscalar vector currents in the proton states, which in term, allow determination of two combinations of the matrix elements of up, down, and strange quark currents in the proton.

To completely determine the matrix elements of all quarks, one needs an additional combination. We introduce the SU(3) flavor singlet $\lambda^0 = \sqrt{2/3}\mathcal{I}$ operator (where \mathcal{I} is the identity matrix) and define the associated flavor singlet current. The photon does not couple to this current but the weakly interaction neutral gauge boson Z does.

In the standard model, the quark interacts with the neutral Z boson via

$$L = -\frac{g}{4 \cos \theta_W} \sum_i \bar{\psi}_i \gamma^\mu (g_V^i - g_A^i \gamma_5) \psi_i Z_\mu \quad (4.42)$$

where the vector and axial vector coupling is

$$\begin{aligned} g_V^i &= 2(t_{3L}(i) - 2q_i \sin^2 \theta_W) \\ g_A^i &= 2t_{3L}(i) . \end{aligned} \quad (4.43)$$

At tree level, one has $G_F/\sqrt{2} = g^2/8M_W^2$. Therefore, the Z -boson not only interacts with the vector currents of quarks and leptons, but also interacts with the axial vector currents. The quark vector current is

$$\begin{aligned} J_Z^\mu &= \left(1 - \frac{8}{3} \sin^2 \theta_W\right) \bar{u}\gamma^\mu u - \left(1 - \frac{4}{3} \sin^2 \theta_W\right) \bar{d}\gamma^\mu d \\ &\quad - \left(1 - \frac{4}{3} \sin^2 \theta_W\right) \bar{s}\gamma^\mu s \end{aligned} \quad (4.44)$$

whereas the axial current is given by

$$A_Z^\mu = \bar{u}\gamma^\mu \gamma_5 u - \bar{d}\gamma^\mu \gamma_5 d - \bar{s}\gamma^\mu \gamma_5 s . \quad (4.45)$$

One can also define the quark axial currents with flavor structure similar to the vector currents.

Consider now the interaction between the electron and the proton, not only there is a photon exchange but also a Z-boson exchange. At low energy, the Z-boson exchange is very small but can be studied using parity-violation in the scattering of longitudinally polarized electrons.

Suppose the incident electron is polarized in the helicity-1/2 state. Then the cross section will have a correlation $\vec{S}_e \cdot \vec{k}$ where \vec{S}_e is the polarization and \vec{k} is the momentum of the electron. Under parity transformation, the above is a pseudo-scalar. Since the Z-boson violates parity, the above correlation is allowed. If so, one can flip the spin of the electron, find a difference in the scattering cross section. This is termed the left-right asymmetry.

The S-matrix for the Z exchange is

$$\begin{aligned} S &= (2\pi)^4 \delta^4(k + P - P' - k') \bar{u}(k') \left(-i \frac{g}{2 \cos \theta_W} \right) \gamma^\mu (g_V^e - g_A^e \gamma_5) u(k) \\ &\quad \times \frac{-i}{q^2 - M_Z^2} \left(-i \frac{g}{2 \cos \theta_W} \right) \langle P' | J_Z^\mu | P \rangle \\ &= -i (2\pi)^4 \delta(k + P - k' - P') \frac{g^2}{16 \cos^2 \theta_W} \frac{1}{M_Z^2} \bar{u}(k') (g_V^e - g_A^e \gamma_5) u(k) \langle P' | J_Z^\mu | P \rangle. \end{aligned} \quad (4.46)$$

Therefore the invariant amplitude including the photon exchange reads,

$$\mathcal{M} = -\frac{e^2}{Q^2} \bar{u}(k') \gamma^\mu u(k) \langle P' | J_\mu | P \rangle - \frac{G_F}{2\sqrt{2}} \bar{u}(k') (g_V^e - g_A^e \gamma_5) u(k) \langle P' | J_Z^\mu | P \rangle. \quad (4.47)$$

We can square this amplitude to obtain

$$|\mathcal{M}|^2 = \frac{e^2 G_F}{2\sqrt{2} Q^2} \ell^{\mu\nu} H_{\mu\nu}. \quad (4.48)$$

Let us first calculate the lepton tensor. We are only interested in the spin-dependent part of the tensor,

$$\begin{aligned} \ell^{\mu\nu} &= \frac{1}{2} \text{Tr}[\gamma^\mu \not{k}' \gamma^\nu] (g_V^e - g_A^e \gamma_5) (1 + \lambda \gamma_5) \not{k} \\ &= 2\lambda [g_V^e \epsilon^{\mu\nu\alpha\beta} k'_\alpha k_\beta - g_A^e (k'^\mu k^\nu + k'^\nu k^\mu - g^{\mu\nu} k \cdot k')] \end{aligned} \quad (4.49)$$

Because $g_V^e \sim 1 - 4 \sin^2 \theta_W \sim 0$, the electron's neutral current coupling is almost entirely axial. Therefore we can approximate the lepton tensor by just the axial part, which is symmetric. Therefore, let us just consider the symmetric part of the hadron tensor,

$$H^{\mu\nu} = (-q^\mu q^\nu + g^{\mu\nu} q^2) G_M^\gamma G_M^Z + (P + P')^\mu (P + P')^\nu \frac{G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z}{1 + \tau} \quad (4.50)$$

From this we can calculate the asymmetry as,

$$A = \left[\frac{-G_F Q^2}{8\sqrt{2}\pi\alpha} \right] \cdot (A_E + A_M + A_A) \quad (4.51)$$

where the three terms $A_E \sim G_E^\gamma G_E^Z$,

$$A_M = \frac{\epsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z}{\epsilon (G_E^\gamma)^2 + \tau (G_E^\gamma)^2} \quad (4.52)$$

For an order of magnitude estimate,

$$A \sim 10^{-4} Q^2 \quad (4.53)$$

Therefore the experiment is very challenging.

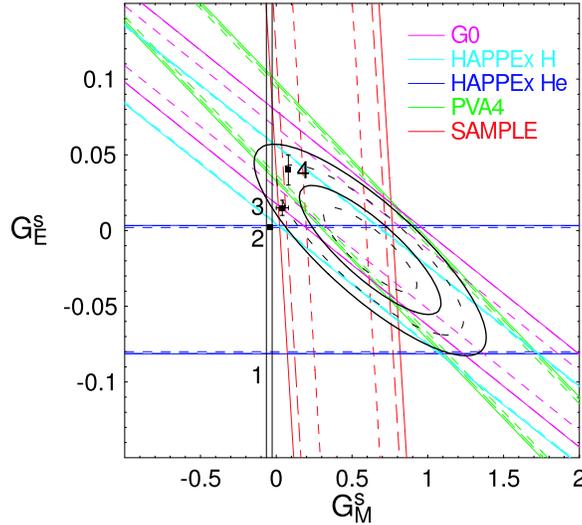


Figure 4.3: Experimental constraints on the strange electric and magnetic form factors of the nucleon at $Q^2 = 0.1 \text{ GeV}/c^2$.

From the expression for the vector electroweak currents, we can find the strange quark matrix elements,

$$G_{EM}^s = (1 - 4 \sin^2 \theta_W^2) G_{EM}^{\gamma,p} - G_{EM}^{\gamma,n} - G_{EM}^{Z,p} \quad (4.54)$$

The strange quark contribution to the magnetic moment of the proton is

$$\mu_s = G_M^s(Q^2) \quad (4.55)$$

Since the nucleon has no net strangeness, $G_M^s(0) = 0$. However, one can define a strange quark radius as

$$r_s^2 = -6 \frac{dG_E^s(Q^2)}{dQ^2}. \quad (4.56)$$

The first experiment to study strange form factors with this method was the SAMPLE experiment, performed at the MIT-Bates linear accelerator. The measurements surprisingly favored a positive result for the strange magnetic form factor G_M^s . This result has been confirmed by subsequent experiments at other laboratories, and the present constraints on G_M^s and G_E^s are shown in Figure 4.3.

4.4 Deep Inelastic Scattering and Parton Model

One way to study the structure of a composite system is to knock out the fundamental constituents and study their energy-momentum distribution. For example, consider the electron in a hydrogen

atom. If one strikes it with some momentum transfer q , by measuring its final momentum \vec{k}' , one can figure out the initial momentum \vec{k} . The cross section will tell us the electron momentum distribution $n(k)$. Another example—in a quantum liquid, like liquid ^4He , the atoms distribute in different momentum states according to $n(k)$, and there is presumably a considerable accumulation of particles at $k = 0$ (Bose-Einstein condensation). Through neutron scattering, one can measure this distribution. A third example—quasi-elastic electron scattering on a nucleus made of neutrons and protons. The scattering electron knocks out a proton or a neutron through exchange of a high-momentum photon.

If there are quarks inside the proton, it would be very useful to know their distributions in momentum space. One way to learn this is to scatter highly virtual photons off the quarks in the proton and measure the distribution. This is called deep-inelastic scattering (DIS), first performed by Friedmann, Kendall, and Tylor et al at Stanford Linear Accelerator Center (SLAC). However, since the quarks cannot exist in isolation, it is a bit tricky to interpret the experiment data.

Let us introduce some terminology first. Consider electron scattering on a proton producing a final state $|X\rangle$. Using one photon exchange approximation, the S -matrix is

$$S = (2\pi)^4 \delta^4(k + P - P' - k') \bar{u}(k') (-ie\gamma^\mu) u(k) \times \frac{-i}{q^2} \langle X | (ie) J^\mu | P \rangle, \quad (4.57)$$

where X is any hadronic final state. The corresponding inclusive cross section is

$$\frac{d\sigma}{d\Omega dE} = \frac{\alpha^2 E'}{Q^4 E} \ell_{\mu\nu} W^{\mu\nu}, \quad (4.58)$$

where $Q^2 = (k' - k)^2$ and the hadron tensor is

$$W_{\mu\nu} = \frac{1}{4\pi} \sum_X \langle P | J_\mu | X \rangle \langle X | J_\nu | P \rangle (2\pi)^4 \delta^4(P + q - P_X) \quad (4.59)$$

Since the final states are summed over, the W tensor depends only on the initial nucleon momentum P and the photon momentum q . According to Lorentz symmetry, parity and time reversal invariance, and current conservation, one can express it in terms of two invariant tensors,

$$W_{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{W_2}{M^2} \left(P^\mu - q^\mu \frac{(P \cdot q)}{q^2} \right) \left(P^\nu - q^\nu \frac{(P \cdot q)}{q^2} \right) \quad (4.60)$$

W_1 and W_2 are functions of two Lorentz scalars, Q^2 and ν .

The early SLAC data indicates that if W_1 and W_2 are plotted as a function of $x = Q^2/2M\nu$, they are nearly independent of Q^2 ! This behavior is called Bjorken scaling. If x is fixed $Q^2 \rightarrow \infty$, this is called the Bjorken limit.

To explain Bjorken scaling, Feynman introduced the so-called parton model in which the nucleon is made of non-interacting partons (quarks), and in DIS, the photon scatters off these free partons. Partons can be any particles with no internal structure. Of course the partons must be interacting because otherwise the nucleon will fall apart. To understand why the partons can be viewed as free in DIS, one can consider a frame in which the nucleon is moving very fast, say, with the speed of light. Suppose the typical interaction time-scale in the proton is τ , in the moving frame, the interaction time becomes $\tau\gamma$, where $\gamma = 1/\sqrt{1-v^2}$ is Lorentz dilation factor. When the speed the proton v approach that of the speed of light 1, the interaction time in the proton is so long that

proton configuration can be considered essentially frozen. Alternatively, in the rest frame of the proton, the photon interaction time is of order $1/Q$, which is much shorter than the typical hadronic interaction time which is order $1/\Lambda_{\text{QCD}}$. Therefore, the physics of scattering can be separated from the bound state physics, and the partons can be considered as essentially free during scattering. This is called factorization in QCD.

Let us calculate the hadronic tensor by summing over scattering on partons

$$W^{\mu\nu} = \int \frac{dx_F}{x_F} f(x_F) w^{\mu\nu} , \quad (4.61)$$

where $x_F P$ is the longitudinal momentum carried by a parton. $f(x_F)$ is the parton density. $w^{\mu\nu}$ is the hadron tensor for a single quark. Taking into account the antiquark contribution,

$$W^{\mu\nu} = -\frac{1}{4\pi} \text{Im} \int dx \text{Tr} \left[\gamma^\mu \frac{x \not{p} + \not{q}}{(xp+q)^2 + i\epsilon} \gamma^\nu \not{p} f_1(x) \right] + \text{crossing} . \quad (4.62)$$

After doing the trace, we have,

$$W^{\mu\nu} = \frac{1}{2\nu} (f_1(x_B) - f_1(-x_B)) (2x_B p^\mu p^\nu + p^\mu q^\nu + p^\nu q^\mu - g^{\mu\nu} \nu) . \quad (4.63)$$

where $-f_1(-x_B)$ is the anti-parton contribution $\bar{f}_1(x_B)$. Comparing this with the definition of the structure functions,

$$\begin{aligned} W_1 &= \frac{1}{2} \sum_i e_i^2 [f_1^i(x_B) + \bar{f}_1^i(x_B)] , \\ W_2 &= \frac{M}{\nu} x_B \sum_i e_i^2 [f_1^i(x_B) + \bar{f}_1^i(x_B)] . \end{aligned} \quad (4.64)$$

where we have restored the summation over quark flavors and included the weight of quark charges. Define the scaling functions,

$$\begin{aligned} F_1(x_B) &= W_1 = \frac{1}{2} \sum_i e_i^2 [f_1^i(x_B) + \bar{f}_1^i(x_B)] , \\ F_2(x_B) &= \frac{\nu}{M} W_2 = x_B \sum_i e_i^2 [f_1^i(x_B) + \bar{f}_1^i(x_B)] , \end{aligned} \quad (4.65)$$

We immediately have the celebrated Callan-Gross relation,

$$F_2(x_B) = 2x_B F_1(x_B) . \quad (4.66)$$

Two comments are in order. First, the scaling functions in the impulse approximation depend only on x_B , not on Q^2 . Therefore, the parton model explains the scaling naturally. Second, in the above derivation no assumptions are made about the quark interactions before scattering. In fact, the same result will be obtained if partons are assumed to be off-shell due to initial state interactions. Therefore Feynman's parton model is only a model for the scattering process, not for the internal dynamics of the nucleon.

Deep-inelastic scattering (electron, muon, neutrino) is one of the most important processes to probe the Feynman momentum distributions of quarks and gluons. There are other hard scattering

processes from which one can learn these distributions as well. For example, the Drell-Yan process in which hadron-hadron scattering producing a lepton pair through quark and antiquark annihilation; Direct photon production, heavy-quark production etc. In the past 30 years, thousands of experimental data have been accumulated; and one can make very general parameterizations of these distributions and fit the parameters to the experimental data. The result is a set of phenomenological parton distributions which has been very useful to characterize the structure of the nucleon and to calculate production of new particles in hadron collisions. The well-known parton distribution sets include CTEQ (USA), GRV (Germany), and MRS (England) distributions.

What one has learn about the nucleon structure through high-energy scattering? First of all, one learn there are indeed 2 up valence quarks and 1 down quark, with electric charge $2/3$ and $-1/3$ of the proton, respectively. Second, the number of quarks is infinite because the integration $\int q(x)dx$ does not seem to converge. This is because there are infinite number of quark and antiquark pairs in the proton. Finally, the gluons have been found to play very important role in the nucleon structure. In fact, by forming the integral $\int dx x q(x)$, one can find the fraction of the nucleon momentum carried by quarks. The experimental data indicate that this is only about 50% or so. The missing momentum must be carried by the gluons. Therefore the charge-neutral gluons play extremely important role in determining the structure of the nucleon.

4.5 Physics of Parton Distributions

In order to develop some intuition for the physical meaning of the structure function $F_2(x)$, we consider some simple toy models of a nucleon. These are illustrated in the set of diagrams and graphs in Figure 4.4. We first note that for a point nucleon that is simply a single quark, F_2 is just a delta function at $x = 1$ (all the momentum is carried by one object). For a nucleon consisting of three quarks at rest (in the nucleon rest frame), each would carry $1/3$ of the momentum in the infinite momentum frame, and F_2 would then be a delta function at $x = 1/3$. For three interacting quarks, we expect a smeared out distribution peaked in the region $x \simeq 1/3$. Finally, for the case of three interacting quarks plus a “sea” of quark-antiquark pairs we expect that the low x region would become populated by soft pairs. (When a quark emits a quark-antiquark pair, all three resulting particles have lower x than the original quark.)

In QCD, the parton distributions can be expressed as the matrix elements of non-local quark operators. The moments of parton distributions can be calculated in lattice QCD. The nucleon models, such as NR quark model and MIT bag model, can be used to calculate the distribution.

The structure functions of the neutron and proton have been studied in great detail and much is known about them. We summarize some of the more important properties here. Let’s begin by writing out F_2 for both nucleons assuming only up and down quarks are present. (One can include strange quarks, but we omit them here for simplicity. Quantitatively, the up and down quarks dominate the structure functions.) To simplify the notation we define the up and down momentum fraction distributions as

$$u(x) \equiv f_u(x) \tag{4.67}$$

$$d(x) \equiv f_d(x) \tag{4.68}$$

and similarly for the antiquarks. Then we can write the structure functions as follows.

$$F_2^p(x) = x \left\{ \frac{4}{9} [u^p(x) + \bar{u}^p(x)] + \frac{1}{9} [d^p(x) + \bar{d}^p(x)] \right\} \tag{4.69}$$

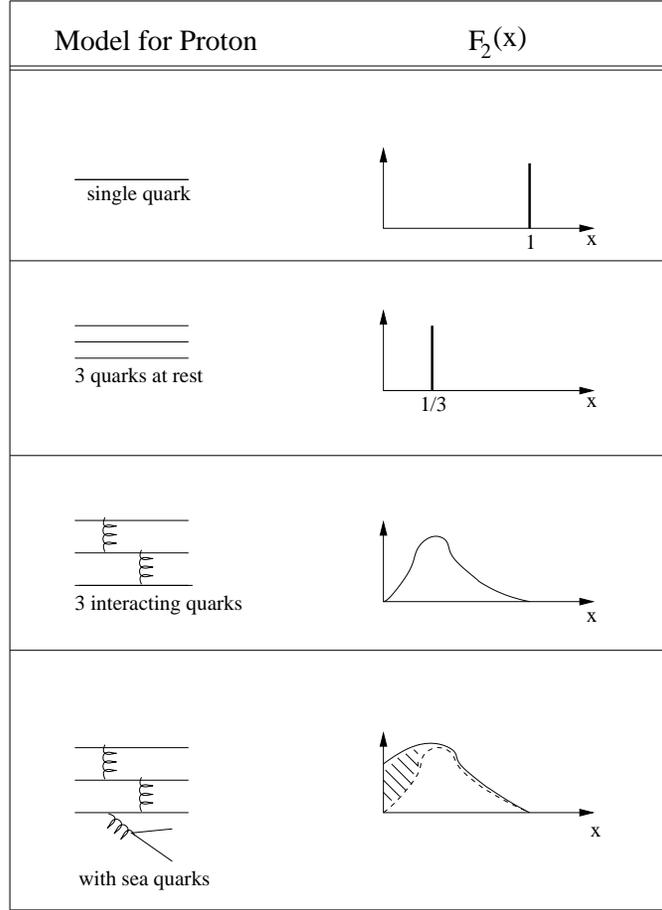


Figure 4.4: Simple nucleon models to illustrate the behavior of the deep inelastic structure functions.

$$F_2^n(x) = x \left\{ \frac{4}{9} [u^n(x) + \bar{u}^n(x)] + \frac{1}{9} [d^n(x) + \bar{d}^n(x)] \right\} \quad (4.70)$$

Under isospin flip, $u \leftrightarrow d$ and $n \leftrightarrow p$. Therefore, we have

$$F_2^n(x) = x \left\{ \frac{4}{9} [d^p(x) + \bar{d}^p(x)] + \frac{1}{9} [u^p(x) + \bar{u}^p(x)] \right\} \quad (4.71)$$

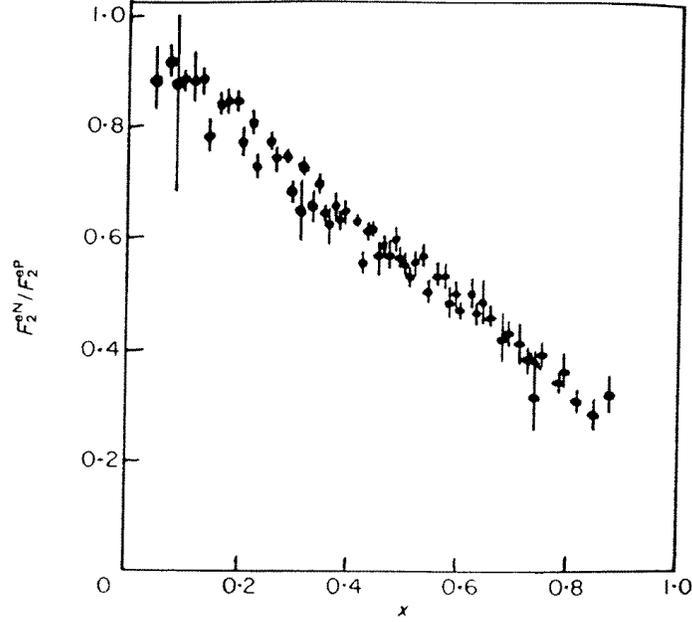
or, defining $u(x) \equiv u^p(x)$, $d(x) \equiv d^p(x)$

$$F_2^p = x \left[\frac{4}{9} (u + \bar{u}) + \frac{1}{9} (d + \bar{d}) \right] \quad (4.72)$$

$$F_2^n = x \left[\frac{4}{9} (d + \bar{d}) + \frac{1}{9} (u + \bar{u}) \right] \quad (4.73)$$

where we have suppressed writing the x -dependence of the functions u and d .

Note that, based on this expression, we expect the inequality $\frac{1}{4} \leq \frac{F_2^n}{F_2^p} \leq 4$ to hold. This is well satisfied by the experimental data as shown in Fig. 4.5 In the region at low $x \ll 1$, the “sea” of $\bar{q}q$

Figure 4.5: Data for F_2^N/F_2^P vs. x .

pairs dominates the structure function and one observes $\frac{F_2^N}{F_2^P} \rightarrow 1$. At higher $x \rightarrow 1$, the “valence” quarks dominate and we find $\frac{F_2^N}{F_2^P} \rightarrow 1/4$ since $u(x) > d(x)$ in the proton (there are 2 valence up quarks vs. only one valence down quark).

In addition to rather direct observation of the quark structure by measuring $F_{1,2}$, it is possible to obtain evidence of the existence of a sea of gluons in the nucleon. The gluons carry a significant fraction of the momentum of the nucleon (in the infinite momentum frame used to analyze deep inelastic scattering) which affects a “momentum sum rule” that indicates the fraction of the momentum carried by the quarks. For scattering from an isoscalar nucleus (like deuterium) we define

$$F_2^N(x) \equiv \frac{1}{2}(F_2^p + F_2^n) \quad (4.74)$$

$$= \frac{5}{18}x[u(x) + \bar{u}(x) + d(x) + \bar{d}(x)] \quad (4.75)$$

and so

$$\frac{18}{5} \int_0^1 F_2^N(x) dx = \int_0^1 x \sum_i f_i(x) dx. \quad (4.76)$$

Thus one can measure the sum of the momentum fractions of all the quarks (including antiquarks) via this integral. If there were no other significant constituents then the above integral should be

unity. However, experimentally we observe

$$\frac{18}{5} \int_0^1 F_2^N(x) dx = 0.50 \pm 0.05, \quad (4.77)$$

a value much less than one. This observation is consistent with about $\frac{1}{2}$ of the nucleon momentum being carried by the gluons.

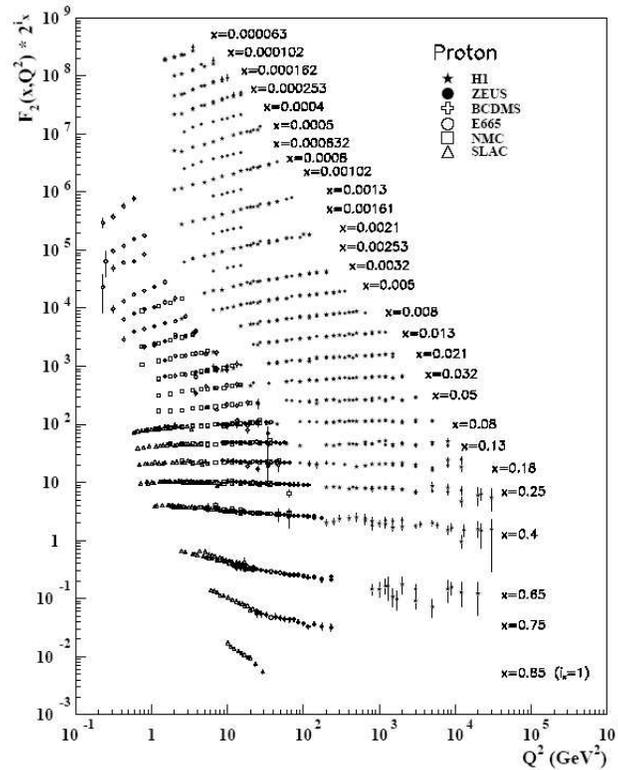


Figure 4.6: Experimental data for the structure function $F_2(x)$ for the proton at various values of x vs. the squared momentum transfer Q^2 .

This interpretation is also supported by the observed mild Q^2 dependence of F_2 (shown in Figure 4.6) due to radiation of gluons by quarks. At finite q^2 there are corrections to the simple parton picture we have developed, and the number of gluons and sea quarks becomes dependent upon the spatial resolution of the virtual photon. At lower Q^2 , the larger spatial region probed by the lower resolution virtual photon effectively contains additional gluons and $\bar{q}q$ pairs. These become resolved at higher Q^2 with the result that there are effectively more partons (quarks, antiquarks, and gluons) carrying the momentum of the nucleon at higher Q^2 . Since the total of the momentum fractions must sum to unity, each parton carries, on average, lower x at higher Q^2 . This leads to the correction for “scaling violation” where the observed structure functions increase with Q^2 at lower x (due to the greater abundance of soft partons), but decrease with Q^2 at higher x (to keep the total momentum sum fixed).

The parton splitting process can be described by the splitting function P_{ij} . Using QCD perturbation theory, one can calculate them. Once we know these functions, one can calculate the evolution of the quark density as a function of Q^2 ,

$$\frac{dq(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \left[\int_x^1 P_{qq} \left(\frac{x}{y} \right) q(y, Q^2) dy + \int_x^1 P_{qg} \left(\frac{x}{y} \right) g(y, Q^2) dy \right] \quad (4.78)$$

This evolution has been tested to high precision.

4.6 Quark Spin Structure of the Nucleon

The constituent quarks considered in the quark models (Chapter 3) of the nucleon are massive ($\sim M_P/3$) objects with the same spin, charge, and magnetic properties of the massless objects that we observe in deep inelastic scattering. While these quark model objects might be related to the degrees of freedom observed in deep inelastic scattering, they are not identical and this confusion has even been the source of what is commonly called the “proton spin crisis”.

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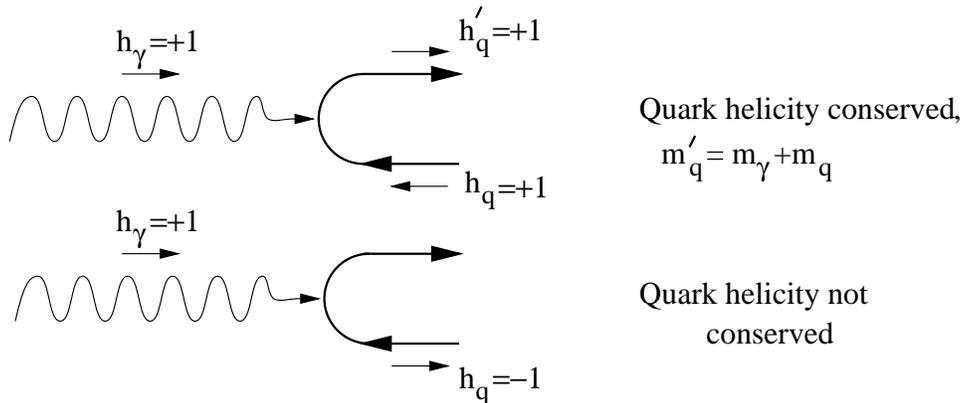


Figure 4.7: Diagram in the Breit frame illustrating how helicity conservation leads to sensitivity of polarized virtual photon amplitudes to the spin structure of the quarks.

The spin of the quarks can be probed in polarized deep-inelastic scattering, in which the lepton and target proton are both polarized along the scattering axis. The polarized electron exchanges a polarized virtual photon (its momentum is almost collinear with that of the initial electron) with the target. Helicity conservation implies that the virtual γ inherits some of the incident lepton helicity, and so we have a virtual γ with some net helicity. Consider the absorption of a very high energy polarized γ on a quark of definite helicity. Recall that for a massless fermion, the electromagnetic interaction will conserve helicity. As shown in Figure 4.7, a + helicity quark can only absorb a + helicity photon. Similarly, a - helicity quark can only absorb a - helicity photon.

Then it is easy to see that by studying the difference under reversal of photon helicity (or equivalently reversing the proton spin), we determine the probability that the struck quark has the same helicity as the incident lepton for a fixed spin orientation of the proton. In particular, the

cross section difference

$$\Delta\sigma = \sigma_{++} - \sigma_{+-} \quad (4.79)$$

is proportional to the combination of polarized momentum fraction distributions (“spin structure functions”):

$$\Delta \propto \frac{1}{2} \sum_i e_i^2 (q_i^+(x) - q_i^-(x)) \equiv g_1(x) \quad (4.80)$$

where $q_i^{+(-)}$ is defined to be the momentum fraction distribution for quark spin parallel (antiparallel) to the nucleon spin.

Suppose the nucleon target is polarized with a spin vector S , then the hadron tensor $W^{\mu\nu}$ will contain terms depending on S . In fact, there are two such terms,

$$W^{[\mu\nu]} = -i\epsilon^{\mu\nu\alpha\beta} q_\alpha \left[G_1(\nu, Q^2) S_\beta / M^2 + G_2(\nu, Q^2) (S_\beta \nu M - P_\beta S \cdot q) / M^4 \right]. \quad (4.81)$$

In the Bjorken limit, we obtain two scaling functions,

$$\begin{aligned} g_1(x, Q^2) &= \left(\frac{\nu}{M} \right) G_1(\nu, Q^2) \rightarrow g_1(x), \\ g_2(x, Q^2) &= \left(\frac{\nu}{M} \right)^2 G_2(\nu, Q^2) \rightarrow g_2(x), \end{aligned} \quad (4.82)$$

which are non-vanishing. The structure function g_1 is extracted from the measured asymmetries of the scattering cross section as the beam or target spin is reversed. These asymmetries are measured with longitudinally polarized beams and longitudinally ($A_{||}$) and transversely (A_{\perp}) polarized targets. We define the differences $\Delta q_i(x) \equiv (q_i^+(x) - q_i^-(x))$, and we expect the measured “asymmetry” to be

$$A = \frac{\sum_i e_i^2 \Delta q_i(x)}{\sum_i e_i^2 q_i(x)} = \frac{g_1(x)}{F_1(x)}. \quad (4.83)$$

Thus, by measurement of this asymmetry one can determine experimental values for the spin dependent structure function $g_1(x)$.

By integrating over x , one forms the proton and neutron integrals

$$I^p = \int_0^1 g_1^p(x) dx = \frac{1}{2} \left(\frac{4}{9} \Delta u + \frac{1}{9} \Delta d \right) \quad (4.84)$$

$$I^n = \frac{1}{2} \left(\frac{1}{9} \Delta u + \frac{4}{9} \Delta d \right) \quad (4.85)$$

where we have defined the integrals

$$\Delta u \equiv \int \Delta u(x) dx \quad (4.86)$$

$$\Delta d \equiv \int \Delta d(x) dx. \quad (4.87)$$

Now, it seems reasonable (at least as a rough estimate) to ignore the antiquarks and take the simple quark model values

$$\Delta u = \frac{4}{3}; \quad \Delta d = -\frac{1}{3} \quad (4.88)$$

which can be obtained from the quark model wave function and the expressions

$$\Delta u = \left\langle \psi_p \left| \sum_i \left[\hat{t}_3(i) + \frac{1}{2} \right] \sigma_z(i) \right| \psi_p \right\rangle \quad (4.89)$$

$$\Delta d = \left\langle \psi_p \left| \sum_i \left[-\hat{t}_3(i) + \frac{1}{2} \right] \sigma_z(i) \right| \psi_p \right\rangle. \quad (4.90)$$

Note that we have already utilized these matrix elements in our magnetic moment calculation

$$\mu_p = \Delta u \mu_u + \Delta d \mu_d \quad (4.91)$$

$$= \frac{4}{3} \mu_u - \frac{1}{3} \mu_d \quad (4.92)$$

and that the fraction of the proton spin carried by the quarks is $(\Delta u + \Delta d) = 1$. These values for Δu and Δd yield the predictions

$$I^p = \frac{5}{18} \cong 0.28 \quad (4.93)$$

$$I^n = 0 \quad (4.94)$$

Experimentally, one finds the results

$$I^p \cong 0.13 \quad (4.95)$$

$$I^n \cong -0.03 \quad (4.96)$$

where the neutron value is indeed small (as expected) but the proton value is very much smaller than predicted by the simplest quark model.

A more careful analysis, including the possible contribution from strange quark-antiquark pairs, can be written as follows:

$$I^p = \int_0^1 g_1^p(x) dx = \frac{1}{2} \left(\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right) \quad (4.97)$$

$$I^n = \frac{1}{2} \left(\frac{1}{9} \Delta u + \frac{4}{9} \Delta d + \frac{1}{9} \Delta s \right) \quad (4.98)$$

$$I^p - I^n = \frac{1}{6} (\Delta u - \Delta d) = \frac{1}{6} g_A \quad (4.99)$$

where g_A is a constant from neutron β decay $n \rightarrow p + e^- + \bar{\nu}_e$. To understand why the above equation is true, one can use the isospin symmetry to rewrite the neutron-proton matrix element in terms of the proton matrix element alone,

$$\langle P | \bar{u} \gamma^\mu \gamma_5 u - \bar{d} \gamma^\mu \gamma_5 d | P \rangle = g_A \bar{U}(p) \gamma^\mu \gamma U(p) \quad (4.100)$$

From this equation, it appears that the matrix element $\bar{u} \gamma^\mu \gamma_5 u$ is related to the the fraction of the spin carried by the quark. Indeed, if the proton is polarized in the z-direction, it can be shown that $\gamma^0 \gamma^3 \gamma^5$ is related to Σ^z , the spin operator for a Dirac particle. The spin operator of a relativistic particle is

$$\vec{S} = \psi^\dagger \frac{\vec{\Sigma}}{2} \psi \quad (4.101)$$

where $\vec{\Sigma}$ is a non-relativistic generalization of the Pauli matrices

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}. \quad (4.102)$$

If we consider the matrix element of the spin operator in the nucleon state at rest, then

$$\langle PS | \vec{S} | PS \rangle = \Delta\psi \vec{s}. \quad (4.103)$$

where \vec{s} is the spin polarization vector of the nucleon in the rest frame. $\Delta\psi$ is then the fraction of the nucleon spin carried by the quark spin.

According to our discussion in the previous chapter, we also know the combination

$$\Delta u + \Delta d - 2\Delta s \quad (4.104)$$

from Σ and Λ beta decay (using SU(3) symmetry). $g_8 = \Delta u + \Delta d - 2\Delta s$. Experimentally, one has $g_8 = 0.67$. Therefore, measurement of I^p or I^n allows one to solve for Δu , Δd , and Δs separately.

The experimental result for the spin carried by the quarks can be summarized as follows:

$$\Delta u + \Delta d + \Delta s = 0.30 \pm .03. \quad (4.105)$$

This is still much smaller than the value close to unity one expects in a simple quark model. More careful use of the quark model including relativistic effects gives lower predictions of about 2/3 for this sum, but still it appears that much of the nucleon spin does not reside in the quark helicity.

It is important to distinguish these quarks studied in deep inelastic scattering from the quark model objects. The ‘‘constituent quarks’’ in the quark model are effective objects with complicated internal structure involving gluons and quark-antiquark pairs. The use of these simple degrees of freedom hides their complex substructure, which is in fact probed in deep inelastic scattering. These gluons and $\bar{q}q$ pairs which are not explicit degrees of freedom in the simple quark model can carry some of the spin (and momentum!) of the nucleon. However, they are counted as part of the constituent quark in the simple quark model.

The question is where is the remainder of the proton spin? First of all, there is the quark orbital angular momentum. Then there is the gluon contribution because the gluons are found to carry about 50% of the momentum of the nucleon. So one can write

$$1/2 = J_q + J_g. \quad (4.106)$$

with $J_q = \Delta\Sigma/2 + L_q$. However, the separation is scale dependent (μ). One can show that in the large scale limit, the gluon can carry as much as 50% angular momentum of the nucleon.

On the other hand, the quark orbital angular momentum can be measured through the so-called deeply virtual Compton scattering. Deeply-virtual Compton scattering is a process in which a high-energy, virtual photon strikes a quark in the proton, the quark immediately radiates a photon and returns to the proton to form a recoil proton. In the process, one measures the so-called generalized parton distribution, which is in fact, related to the quantum phase-space Wigner distribution.

We know that form factors describe the spatial distributions of charge and current, and the parton distributions measured in DIS describe the momentum space distribution. A more complete information is a combined coordinate and momentum space distribution, which is a Wigner

distribution. Consider a one-dimensional quantum mechanical system with wave function $\psi(x)$. One can define a phase space distribution

$$W(x, p) = \int d\xi e^{i\xi p} \psi^*(x - \xi/2) \psi(x + \xi/2) . \quad (4.107)$$

After integrating over p , one gets a coordinate space density distribution. After integrating over x , one gets a momentum space density distribution. In general, we have a quantum phase space distribution. Deeply-virtual Compton scattering measures such a distribution which combines the form factors and Feynman parton distribution information.

4.7 Problems

1. In elastic electron scattering, calculate the cross section in terms of electric and magnet form factor G_E and G_M .
2. Derive the magnetic moment in term of the form factors of the electromagnetic current.
3. Work out the structure function W_1 and W_2 in parton model, as shown in Eqs. (4.64)
4. Calculate the cross section asymmetry $\Delta\sigma$ in polarized DIS in terms of the scaling function $g_1(x)$ and $g_2(x)$.
5. Calculate Δu and Δd in MIT bag model.