

Weak and Electromagnetic Interactions.

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Summary. — The postulate of a « local connection » in a [3] charge space leads to the introduction of three spin one fields. One of these can be identified with the electro-magnetic field and the other two can be shown to mediate all known weak interactions, thus unifying these interactions with electro-magnetism. The theory takes account of the fact that weak interactions violate parity and strangeness conservation while electro-magnetic interactions do not do so.

1. — D'ESPAGNAT, PRENTKI and SALAM ⁽¹⁾ have recently advanced the hypothesis that a three-dimensional [3] charge space Q exists and that all elementary particles correspond to its scalar or vector representations.

These authors show that possibly weak interactions exhibit full rotational symmetry in Q space. Electro-magnetic and strong interactions violate the full symmetry; however these interactions are invariant for rotations around one particular axis in this space (the « charge-axis ») so that Q_z is always conserved.

In this paper we wish to make a fresh approach with the idea of a [3] charge space. Following some ideas first advanced by SCHWINGER ⁽²⁾, we show

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(¹) B. D'ESPAGNAT, J. PRENTKI and A. SALAM: *Nucl. Phys.*, **5**, 447 (1958). This paper will here by referred to as ESP. Similar ideas have been suggested by G. TAKEDA: to be published. In ESP the Q space is designated as M space.

(²) J. SCHWINGER: *Ann. of Phys.*, **2**, 407 (1957).

that parity-conserving weak and electromagnetic interactions taken together form a unit which exhibits full rotational symmetry in Q space. This full symmetry is broken on the one hand by parity-violating weak interactions (γ_5 -invariance) and on the other hand by strong interactions. We go further in respect of the rotational invariance of electro-magnetic and (parity conserving) weak interactions in Q space; we wish to make the hypothesis that the orientation of all three charge axes can be chosen arbitrarily at all space time points.

Just as it is necessary to introduce the electro magnetic field if it is assumed that the orientation of axes in the conventional [2] ⁽³⁾ space spanned by fields Φ and Φ^* (or alternatively by the fields Φ_1 and Φ_2 , where $\Phi = \Phi_1 + i\Phi_2$) is arbitrary at all space time points, our « three-dimensional gauge invariance » makes it necessary to introduce in Q space a triplet of fields consisting of two charged vector bose fields in addition to the electro-magnetic field ⁽⁴⁾. It will be shown that the form of the interaction of the charged fields is such that they can mediate weak interactions.

2. - Let us first consider the problem of the generalized gauge transformation ⁽⁵⁾. Let Ψ represent a vector in charge-space, ($\Psi = \psi_1, \psi_2, \psi_3$ where, ψ_1, ψ_2, ψ_3 are 3 Fermi fields), which transforms as

$$(1) \quad \Psi' = S\Psi .$$

If S is a function of x, y, z, t it is necessary for invariance, that all derivatives of ψ appear in the combination

$$(2) \quad (\partial_\mu - i\varepsilon B_\mu)\psi ,$$

where B_μ are 3×3 matrices, and transform as

$$B'_\mu = S^{-1}B_\mu S + \frac{i}{\varepsilon} S^{-1} \frac{\partial S}{\partial x_\mu} .$$

⁽³⁾ A priori, it is more attractive to associate charge with a [3] group structure, rather than with a [2] structure as in conventional theory. This is because infinitesimal matrices describing rotations in three dimensions possess three eigenvalues $\pm 1, 0$ (rather than just ± 1), so that the same formulation can give charge values of *all* charged and neutral particles. We are indebted for this remark to Prof. M. FIERZ.

⁽⁴⁾ This was first done by SCHWINGER (ref. ⁽¹⁾). The difference between our work and Schwinger's is that here the charged Bose fields arise naturally from the 3-dimensional charge-gauge and there is little arbitrariness in writing down the interaction Lagrangian.

⁽⁵⁾ Mathematically the work in this Section is completely analogous with that of YANG and MILLS (*Phys. Rev.*, **96**, 191 (1954)); and R. SHAW (Cambridge dissertation, unpublished (1954)).

YANG and MILLS were considering the possibility of a generalized gauge transformation in three-dimensional isotopic rather than charge-space.

If Q_i are the set of three rotation matrices in Q space, one may write

$$B_\mu = A_\mu(x) \cdot Q$$

and define

$$(3) \quad f_{\mu\nu} = e_{\mu\nu} - S_{\mu\nu},$$

where

$$(4) \quad e_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu},$$

$$(5) \quad S_{\mu\nu} = \varepsilon(A_\mu \times A_\nu).$$

Here \times is the symbol for vector-product.

It is easy to verify that

$$F_{\mu\nu} = f_{\mu\nu} \cdot Q$$

transforms as

$$(6) \quad F'_{\mu\nu} = S^{-1} F_{\mu\nu} S.$$

The A field (or alternatively $f_{\mu\nu}$ field) has arisen as a direct consequence of our demand that S depends on x, y, z, t .

A charge-gauge invariant Lagrangian for A field is

$$(7) \quad -\frac{1}{4} f_{\mu\nu} f_{\mu\nu}.$$

The $S_{\mu\nu}$ terms are necessary for the invariance. Analogously to ref. (5), the total lagrangian density is

$$(8) \quad -\frac{1}{4} f_{\mu\nu} f_{\mu\nu} - \bar{\Psi} \gamma (\gamma - i\varepsilon Q A) \Psi - m \bar{\Psi} \Psi.$$

Notice that

$$J = \bar{\Psi} Q \Psi = -i(\bar{\Psi} \times \Psi)$$

so that the interaction term in (8) is

$$J \cdot A = \varepsilon \bar{\Psi}_\Lambda \gamma_\mu \Psi \cdot A.$$

It is easy to verify that the subsidiary condition

$$(9) \quad \frac{\partial A_\mu}{\partial x_\mu} = 0,$$

is compatible with the equations of motion derived from the Lagrangian; also that the « current »

$$(10) \quad \mathcal{F}_\mu = \mathbf{J}_\mu + \varepsilon \mathbf{A}_r \times \mathbf{f}_{\mu r}$$

satisfies the equation of continuity

$$(11) \quad \frac{\partial \mathcal{F}_\mu}{\partial x_\mu} = 0.$$

Thus

$$Q = \int \mathcal{F}_4 d^3x,$$

is independent of time.

One can verify that components of \mathbf{A} field carry charge $+\varepsilon$, 0 , $-\varepsilon$. Picking out an arbitrary charge axis and defining

$$(12) \quad A^\pm = \frac{1}{\sqrt{2}}(A_1 \mp iA_2), \quad A^0 = A_3,$$

and similarly for ψ^\pm , ψ^0 we note that

$$\begin{aligned} \frac{1}{2}(f_{\mu r}^1 f_{\mu r}^1 + f_{\mu r}^2 f_{\mu r}^2) &= (e_{\mu\nu}^+ - S_{\mu\nu}^+)(e_{\mu\nu}^- - S_{\mu\nu}^-) = \\ &= \left[\left(\frac{\partial}{\partial x_r} - i\varepsilon A_r^0 \right) A_\mu^+ - \left(\frac{\partial}{\partial x_\mu} - i\varepsilon A_\mu^0 \right) A_r^+ \right] \left[\left(\frac{\partial}{\partial x_r} + i\varepsilon A_r^0 \right) A_\mu^- - \left(\frac{\partial}{\partial x_\mu} + i\varepsilon A_\mu^0 \right) A_r^- \right], \end{aligned}$$

while

$$-\frac{1}{4} f_{\mu r}^3 f_{\mu r}^3 = -\frac{1}{4} e_{\mu\nu}^0 e_{\mu\nu}^0 + \frac{1}{2} S_{\mu\nu}^0 e_{\mu\nu}^0 - \frac{1}{4} S_{\mu\nu}^0 S_{\mu\nu}^0.$$

Identifying A_μ^0 and $e_{\mu\nu}^0$ with the electro-magnetic field, the lagrangian $-\frac{1}{4} \mathbf{f}_{\mu\nu} \mathbf{f}_{\mu\nu}$ is a sum of terms which represent

- 1) the free Maxwell Lagrangian $(-\frac{1}{4} e_{\mu\nu}^0 e_{\mu\nu}^0)$;
- 2) the terms: $-\frac{1}{2}(\partial_\nu^+ A_\mu^+ - \partial_\mu^+ A_\nu^+)(\partial_\nu^- A_\mu^- - \partial_\mu^- A_\nu^-)$,

where

$$\partial^\pm = \left(\frac{\partial}{\partial x} \mp i\varepsilon A^0 \right).$$

These terms describe the conventional electro-magnetic interaction of charged fields A^+ , A^- .

3) Terms $-\frac{1}{4}S_{\mu\nu}^0 S_{\mu\nu}^0 = \frac{1}{4}\varepsilon^2((A_\mu^+ A_\nu^- - A_\mu^- A_\nu^+)^2$ which are in a sense « mass » terms for the charged fields A^+ , A^- .

$$4) \text{ A term } \frac{1}{2}S_{\mu\nu}^0 e_{\mu\nu}^0 = -(i/2)(A_\mu^+ A_\nu^- - A_\nu^+ A_\mu^-)e_{\mu\nu}^0.$$

This term would represent a (Pauli) anomalous magnetic moment for A^+ , A^- fields, if these fields had mass.

The A field by itself is then a most remarkable field. One of its components can be identified with the Maxwell field, the other two represent charged particles, carrying « anomalous magnetic moment » while in place of the conventional mass term $-\mu^2 A^+ A^-$, there is a more complicated (quartic) term

$$\frac{\varepsilon^2}{2}[(A_\mu^+ A_\mu^-)^2 - (A_\mu^+)^2 (A_\mu^-)^2].$$

Even in the presence of interactions with other particles the supplementary condition has the transparent form

$$\frac{\partial A_\mu}{\partial x_\mu} = 0.$$

The theory is renormalizable; it is perhaps the only theory of charged vector mesons which can be renormalized. Finally note that

$$-\bar{\Psi} \times \Psi \cdot A = (\bar{\psi}^+ \psi^+ - \bar{\psi}^- \psi^-)A^0 + (\bar{\psi}_0 \psi^- - \bar{\psi}^+ \psi_0)A^+ + \text{h. c.}$$

3. - Let us assume that $L_1 = (e^+, \nu, e^-)$ form a vector in charge-space. With

$$e^\pm = \frac{1}{\sqrt{2}}(\psi_1 \mp i\psi_2), \quad \psi_3 = \nu,$$

(where the three ψ are Majorana fields), the derivative term in the free Lagrangian is

$$\bar{\Psi} \gamma \partial \Psi,$$

which leads to an interaction with A field of the type $\bar{\Psi} \times \Psi \cdot A$. Writing this in full

$$(14) \quad [\bar{e}^+ \gamma_\mu \nu - \bar{\nu} \gamma_\mu e^-]A_\mu^+ + \text{h. c.} + [\bar{e}^+ \gamma_\mu e^+ - \bar{e}^- \gamma_\mu e^-]A_\mu^0.$$

The last term represents electromagnetic interaction while the first two

terms could give the weak interactions of (e, ν) pair. However so far there is nothing in the theory to pick out any one axis in charge-space.

We wish now to do precisely this, by requiring that the neutrino Lagrangian remain invariant for

$$(15) \quad \nu \rightarrow \gamma_5 \nu .$$

in so far as the neutrino mass must be zero. One result of this is that the first term in L_{int} reads

$$\frac{1}{2}[\bar{e}^+ \gamma_\mu (1 + \gamma_5) \nu - \bar{\nu} \gamma_\mu (1 + \gamma_5) e^-] A_\mu^+ .$$

Thus the neutrino-gauge, conflicting as it does with the [3] charge-gauge, picks an axis in charge-space, and (incidentally) leads to parity violation of interactions of the A_μ^+ , A_μ^- fields. The possibility of gauging the neutrino ($\nu \rightarrow \gamma_5 \nu$) arises from the fact that neutrino has zero mass while the electron does not. In a sense then the existence of mass is incompatible with full rotational symmetry in [3] charge-space; the fact that all known charged particles have mass is the expression of the fact that one particular axis is preferentially chosen in [3] charge-space.

One can consider a second lepton vector $L_2 = (\mu^+, \nu', \mu^-)$, where ν' is gauged as $\nu' \rightarrow -\gamma_5 \nu'$ (*). The interaction Lagrangian for L_2 fields is $-\varepsilon \bar{L}_2 \times L_2 \cdot A$. Through the intermediacy of the A^+ , A^- fields, one then has a possibility of $\mu \rightarrow e + \nu + \nu'$, all helicities given by our interaction Lagrangian being precisely the ones which are observed.

The major problem that remains is the problem of the mass of A^\pm fields. Before the introduction of the parity violating term in (14), there was nothing in the theory to differentiate A^\pm from A^0 . The charged A fields would (in spite of the term

$$-\frac{\varepsilon^2}{2} [(A_\mu^+ A_\mu^-)^2 - (A_\mu^+)^2 (A_\mu^-)^2],$$

acquire no self-mass from its interactions (**). The parity violating terms give the possibility that self-mass for A^+ , A^- no longer vanishes. Also the charge-renormalization for A^+ , A^- interactions is different than for the A^0 interactions.

(*) In a sense ν and ν' are twin neutrinos.

(**) This has actually been checked to the lowest order by S. KAMEFUCHI. We are indebted for a private communication. He has also checked that an effective mass will appear for the A^\pm fields if parity-violating terms are added.

We propose to come back to this problem in a subsequent paper. Here we simply note that if $\varepsilon^2/4\pi = 1/137$, A^+ , A^- fields would need to have an effective mass $\approx \sqrt{\varepsilon^2/g}$, *i.e.* about $30 m_p$. Note that a (quadratic) self-mass is $\alpha\varepsilon^2\Lambda^2$, where Λ is the cut-off mass. Thus $\varepsilon^2\Lambda^2 \approx \varepsilon^2/g$ gives $\Lambda \approx 350 m_p$. This does not appear too excessive a value for the cut-off.

4. - Baryons and mesons.

For π and K mesons, define the charge space vectors $\boldsymbol{\pi}$ and \mathbf{K} composed from (π^+, π^0, π^-) and (K^+, K^0, K^-) particles. (More precisely $\boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3)$

$$\pi^0 = \pi^3, \quad \pi^\pm = \frac{1}{\sqrt{2}}(\pi^1 \mp i\pi^2), \quad K_1^0 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0).$$

The interaction with the A field is

$$-\varepsilon(\partial\boldsymbol{\pi} \times \boldsymbol{\pi} \cdot \mathbf{A}) - \varepsilon^2[\mathbf{A}^2\boldsymbol{\pi}^2 - (\mathbf{A} \cdot \boldsymbol{\pi})^2]$$

and similarly for the K mesons.

For baryons, we make the fundamental assumption that all bare baryon masses are equal. Defining

$$Y_1 = \frac{1}{\sqrt{2}}(A^0 + \Sigma), \quad Y_2 = \frac{1}{\sqrt{2}}(A^0 - \Sigma^0),$$

we write two baryon vectors

$$\mathbf{B}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} p + \Sigma^- \\ i(p - \Sigma^-) \\ n + Y_1 \end{pmatrix}, \quad \mathbf{B}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \Sigma^+ + \Xi^- \\ i(\Sigma^+ - \Xi^-) \\ Y_2 + \Xi^0 \end{pmatrix}.$$

The free baryon Lagrangian can be written as

$$[\bar{\mathbf{B}}_1 D\mathbf{B}_1 + \bar{\mathbf{B}}_2 D\mathbf{B}_2 + \bar{B}_{s1} D B_{s1} + \bar{B}_{s2} D B_{s2}] + \\ + [\bar{\mathbf{B}}_1^c D\mathbf{B}_1^c + \bar{\mathbf{B}}_2^c D\mathbf{B}_2^c + \bar{B}_{s1}^c D B_{s1}^c + \bar{B}_{s2}^c D B_{s2}^c].$$

Here

$$B_{1s} = \frac{1}{\sqrt{2}}[n - Y_1], \quad B_{2s} = \frac{1}{\sqrt{2}}[Y_2 - \Xi^0],$$

D is the Dirac operator and \mathbf{B}^c is the charge-conjugate vector. (Using Majorana representation of Dirac matrices if $\psi = \psi_1 + i\psi_2$ the charge conjugate spinor is $\psi^c = \psi_1 - i\psi_2$).

We note here an essential difference from the case of leptons. Each one

of our vectors \mathbf{B}_1 , \mathbf{B}_2 , etc., contains complex fields as components. For leptons ($\mathbf{L}_1, \mathbf{L}_2$) as well as for π and \mathbf{K} mesons, the corresponding charge-space vectors were real. The reason why $\mathbf{B}_1, \mathbf{B}_2$ are complex lies in baryon-conservation. Thus one can adopt two completely different attitudes for baryons; one can consider that there are essentially four independent charge-space vectors $\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_1^c$ and \mathbf{B}_2^c and baryon-conservation demands that $\mathbf{B}_1, \mathbf{B}_2^c$ (for example) should occur symmetrically. Alternatively one may split each of the vectors $\mathbf{B}_1, \mathbf{B}_2$ into real and imaginary parts and work with the resulting four vectors. It is crucial that there are four baryons which are charged (\mathbf{P}, Σ^-), (Σ^+, Ξ^-). One could never arrive at the ideas of charge-space vectors before the discovery of new particles.

A charge space rotation will produce the interaction Lagrangian.

$$(12) \quad \varepsilon[\bar{\mathbf{B}}_1 \mathbf{B}_1 \cdot \mathbf{A} + \bar{\mathbf{B}}_2 \mathbf{B}_2 \cdot \mathbf{A}] - \varepsilon[\bar{\mathbf{B}}_1^c \mathbf{B}_1^c + \bar{\mathbf{B}}_2^c \mathbf{B}_2^c] \cdot \mathbf{A}.$$

The Lagrangian combines formally electro-magnetic interactions, as well as interaction terms which could be responsible (through A^+, A^- fields) for β decays as well as for hyperon decays. However at this stage only parity conserving weak decays are included.

On the other hand we know from our work with leptons, that the postulated γ_5 -invariance of neutrino interactions picks out a direction in charge-space and also symmetrizes the weak lepton interactions for parity-conserving and parity-violating terms. It thus appears that neutrino γ_5 -invariant Lagrangian is also γ_5 -symmetrized in the sense used earlier by SALAM (*).

A term in a Lagrangian is « γ_5 -symmetrized» if the transformation $\psi \rightarrow \gamma_5 \psi$, $m \rightarrow -m$ are made *independently* for all fields concerned (**) and the result summed. Since the lepton and A field interaction Lagrangian happens to be « γ_5 -symmetrized» in the above generalized sense, it seems necessary to « γ_5 -symmetrize» the baryon-Lagrangian as well. Thus

$$(13) \quad p, Y_1, \Sigma^- \rightarrow \gamma_5(p, Y_1, \Sigma).$$

A priori (Σ^+, Y_2, Ξ^-) could transform as $+\gamma_5$ or $-\gamma_5$ but Y_1 and Y_2 are related and thus it appears likely that (Σ^+, Y_2, Ξ^-) $\rightarrow \gamma_5(\Sigma^+, Y, \Sigma^-)$. After this symmetrization the electro-magnetic interaction remains unchanged, (e.g. $\bar{p}\gamma_\mu p \rightarrow \frac{1}{2}(\bar{p}\gamma_\mu p - \bar{p}\gamma_5\gamma_\mu\gamma_5 p) = \bar{p}\gamma_\mu p$, while $\bar{p}\gamma_\mu n \rightarrow \frac{1}{2}(\bar{p}(1 - \gamma_5)n)$).

(*) A. SALAM: *Phys. Rev. Letters* (to be published). « γ_5 -symmetrization» is not synonymous with « γ_5 -invariance» of a Lagrangian though these may be equivalent in some special cases.

(**) If spin zero Bose fields are present $\varphi \rightarrow -\varphi$ simultaneously with $\psi \rightarrow \gamma_5 \psi$. All currently accepted strong and weak Lagrangians appear to be γ_5 -symmetrized.

It is now necessary to introduce strong interactions of baryons and mesons and to insure that these conserve charge in the conventional sense. This is an important point, for we do not wish to use the vector combinations \mathbf{B}_1 and \mathbf{B}_2 (and singlets B_{s1}, B_{s2}) etc., for writing strong interactions. If we did so, and if strong interactions were rotation invariant in the [3] Q space, there would be no distinction between these interactions and weak and e.m. interactions so far as conventional strangeness, isotopic-spin etc., are concerned. In other words it is necessary to state some criterion which guarantees that even if strong interactions are not rotation-invariant in charge-space these conserve charge in the conventional sense.

Such a criterion is easily stated. Among the \mathbf{A} fields, γ_5 symmetry has picked out the component A^0 . The divergence of the current $\mathcal{F}_\mu^{(0)}$ vanishes.

Here $\mathcal{F}_\mu^{(0)}$ equals the third component of

$$\varepsilon(\bar{\mathbf{L}}_1\mathbf{L}_1 + \bar{\mathbf{L}}_2\mathbf{L}_2 + \bar{\mathbf{B}}_1\mathbf{B}_1 + \bar{\mathbf{B}}_2\mathbf{B}_2 - \bar{\mathbf{B}}_1^c\mathbf{B}_1^c - \bar{\mathbf{B}}_2^c\mathbf{B}_2^c) + \\ + \text{terms from } \pi, \mathbf{K} \text{ and } A \text{ fields. (See Equ. (10)).}$$

Thus charge operator

$$Q = \int \mathcal{F}_4^0 d^3x,$$

can be defined in the usual manner. If we now demand that the hamiltonian for all strong interactions H_s must commute with Q (and must satisfy baryon conservation) charge in the conventional sense is conserved even if H_s is not rotation-invariant in Q -space. Thus the usual isotopic rotation invariant Lagrangian with 8 different coupling constants

$$g_1\bar{N}N\pi + g_2\bar{A}\Sigma\pi + g_3\bar{\Sigma}\Sigma\pi + \dots g_5\bar{N}K\Lambda + g_6\bar{N}K\Sigma + \dots$$

is fully acceptable. The mass degeneracy of baryons in the free Lagrangian will be removed by these strong interactions.

5. - The simple interaction Lagrangian written symbolically as

$$(\bar{\mathbf{L}}\mathbf{L} + \bar{\mathbf{B}}\mathbf{B} + \pi\pi + \mathbf{K}\mathbf{K} + \mathbf{A}\mathbf{A}) \cdot \mathbf{A}$$

combines electro-magnetic interactions, as well as weak interactions. For weak interactions γ_5 -symmetrization gives parity-violating terms while electro-magnetic interaction terms still conserve parity. The weak interactions appropriately violate strangeness conservation while electromagnetic interactions do not.

There are also a number of conservation (eq. (10)) laws for the weak inter-

action currents. These are similar to the conservation laws previously discovered by GELL-MANN (*) and ZELDOVICH and GERŠTEJN.

The idea that weak interactions and electromagnetic interactions should be combined, originating as it did with SCHWINGER (2), has been often discussed privately, as has also the possibility that the resulting A -fields should be associated with a Yang-Mills type of gauge transformation. In this connection we are unable to disentangle the extent to which we are indebted to others. Particularly we would like to mention Dr. S. L. GLASHOW who has expressed similar ideas to us privately and a recent preprint (H. MAYER, Dubno) where a future paper on this subject is promised.

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(*) M. GELL-MANN: *Phys. Rev.*, to be published. S. S. GERŠTEJN and ZELDOVICH: *Žurn. Eksper. Teor. Fiz.*, **29**, 698, (1958).

RIASSUNTO (*)

Il postulato di una «connessione locale» in un [3] spazio della carica conduce all'introduzione di tre campi di spin uno. Uno di questi può identificarsi col campo elettromagnetico e si può dimostrare che gli altri due sono il veicolo di tutte le interazioni deboli note, unificando così tali interazioni coll'elettromagnetismo. La teoria tien conto del fatto che, a differenza delle interazioni elettromagnetiche, le interazioni deboli violano la conservazione della parità e della stranezza.

(*) Traduzione a cura della Redazione.