

# RED SHIFT IN THE SPECTRUM OF 40 ERIDANI B\*

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## ABSTRACT

Measurement of 42 hydrogen lines on 27 spectrograms of 40 Eri B leads to an observed red shift,  $\Delta\lambda/\lambda$ , of  $7 \times 10^{-5}$ , corresponding to a velocity of 21 km/sec. This is to be compared to the value of 17 km/sec predicted on the basis of general relativity from the mass and radius of the star. The difference between observed and predicted values is less than the sum of their uncertainties, which are considerable fractions of the values themselves.

## INTRODUCTION

The available observational material concerning displacements of lines in spectra of white-dwarf stars of known mass is meager and uncertain. Adams (1925; Strömberg 1926) on the basis of four spectrograms and Moore (1928) on the basis of 3 spectrograms have published observations of Sirius B, giving a red shift of 20 km/sec. Although the well-known observational difficulties caused by the light of Sirius A are considerable, the agreement between the results of the two experienced observers seems to indicate a reliable determination. Recently, however, lack of confidence has been expressed (e.g., Einstein 1950;<sup>1</sup> Gamow and Critchfield 1949) in the validity of this result, partly because of theoretical difficulties (e.g., Mestel 1952). On the basis of two spectrograms with lines of poor quality, Münch (1944) has obtained a velocity giving a red shift of  $22 \pm 9$  km/sec for the white dwarf 40 Eri B.

It appears desirable to obtain a more nearly definitive determination of the red shift for a white dwarf. Much the most favorable case is 40 Eri B. After Sirius B, it is the brightest known white dwarf (9.5 mag.). There is no danger of contamination of its spectrum from the light of 40 Eri A or C, and the radial velocity of the system is known from observations of A, which has a proper motion and parallax in common with the binary system BC.

## OBSERVATIONS

Between December, 1949, and January, 1953, 37 prismatic spectrograms of 40 Eri B were obtained at Mount Wilson, 22 with the 60-inch reflector (dispersion 78 Å/mm at  $H\gamma$ ) and 15 with the 100-inch (dispersion 67 Å/mm at  $H\gamma$ ). A number of the 60-inch observations were made by Dr. E. C. Yowell, whose assistance is gratefully acknowledged. Exposure times of  $1\frac{1}{2}$ –4 hours on baked Kodak IIa-O plates were employed in an effort to fill in the wings of the very broad hydrogen lines. In this way the relatively narrow cores (Kuiper 1935; Babcock 1948) stand out in enhanced contrast. There is no possibility that light from either 40 Eri A (4.5 mag., K1) or 40 Eri C (11.0 mag., M6) contaminates the spectrum of the white dwarf. Although star A is 4 mag. brighter in the blue and has numerous sharp lines in its spectrum, its distance of 82'' places it safely out of the way. Star C is more than 4 mag. fainter than star B in the blue, and nearly 7'' away in a position angle near 180°, which is perpendicular to the slit length. No trace of such a strong line in the spectra of stars A and C as *Ca* I 4226 could be detected in any of the spectrograms of B.

\* This investigation was carried out under a co-operative arrangement with the Mount Wilson and Palomar Observatories.

<sup>1</sup> "The shifting of spectral lines toward the red [predicted by general relativity theory] . . . is not yet confirmed" (passage written in 1948).

In addition to the spectrograms of 40 Eri B, on each night of observation a plate was taken of 40 Eri A, and, starting in January, 1951, of one or both of the standard velocity stars, HD 36591 (5.3 mag., B2) and  $\eta$  Leo (3.6 mag., A0). These latter stars were chosen to test for possible systematic errors in velocities obtained from measures of hydrogen lines.

The spectrograms of all four stars were measured with a magnification of 14 times. Each spectrogram was measured twice, and the mean displacement obtained for each measured line. Only the hydrogen lines,  $H\delta$ ,  $H\gamma$ , and  $H\beta$ , could be measured in the spectrum of 40 Eri B. At the time of measurement each line was assigned a weight according to the reliability of its appearance. Mean displacements for the different lines and for each of the two sets of plates (60-inch and 100-inch) are given for 40 Eri B in Table 1. The second group of values, being weighted means of only the better lines, is considered the more reliable and is adopted. The other is included to show the effects of choice of lines. The adopted results are obtained from sixteen 60-inch and eleven 100-inch spectrograms.

TABLE 1  
MEASURED DISPLACEMENTS OF LINES IN SPECTRUM OF 40 ERI B

TELE- SCOPE	LINE	ALL LINES				BETTER LINES			
		No. of Plates	Sum of Weights	Mean Displ. (Km/Sec)	P.e. of Mean (Km/Sec)	No. of Plates	Sum of Weights	Mean Displ. (Km/Sec)	P.e. of Mean (Km/Sec)
60-inch . . . . .	$H\delta$	16	4.6	-13	5	3	1.0	-7	8
	$H\gamma$	21	10.6	-22	4	13	5.2	-26	4
	$H\beta$	22	8.4	-18	6	6	3.0	-31	4
100-inch . . . . .	$H\delta$	8	3.1	-12	4	3	1.2	-15	5
	$H\gamma$	13	7.4	-26	3	10	6.0	-25	4
	$H\beta$	11	5.6	-17	6	7	3.8	-22	5

The probable error of the velocity from a single line of unit weight corresponds, on the average, to a displacement of 0.0022 mm on the plate. This value may be compared with the weighted mean difference between two measures of the same line, which is 0.0017 mm. For the sharper lines of the standard stars, the mean difference is 0.0011 mm. These results indicate that the scatter of measured displacements can be adequately accounted for by accidental errors of measurement. The small number of good lines in the case of  $H\delta$  is a result of the small number of plates on which this line shows a core comparable with those of  $H\gamma$  and  $H\beta$ . Part, though apparently not all, of this effect results from the generally weaker exposures at shorter wave lengths.

Since we are interested in the apparent velocity of 40 Eri B relative to the center of mass of the system BC, differential velocities with respect to 40 Eri A, which shows a common parallax and proper motion, should suffice. The hydrogen lines in the spectrum of 40 Eri A, however, are badly blended on plates of the dispersion used, so that direct differences of velocity could not be evaluated. Furthermore, appreciable systematic differences of velocity among the fifteen lines measured in the spectrum of 40 Eri A were found when the wave lengths recommended by the Mount Wilson spectroscopy division were used. Hence I feel that an apparent difference in velocity between stars A and B can best be evaluated by adopting the known velocity of A and investigating possible systematic effects in the measures of B. Spectrograms of HD 36591 and of  $\eta$  Leo, which have strong hydrogen lines and well-determined velocities in the Lick system, were used for the purpose. The corrections to measured velocities indicated by the measures of these

spectrograms are shown in Table 2. Only results from lines of reliable appearance are included. The systematic corrections are disappointingly large. In Table 3 we give the final results of the measurements, both corrected and uncorrected. The weights listed are those of the adopted displacements of Table 1 adjusted, for each telescope, for differences in linear displacement on the plate corresponding to 1 km/sec of velocity.

Giving low weight to the uncorrected values, I finally adopt for the apparent velocity of 40 Eri B,  $-21$  km/sec. A computation of the probable error of this result, when each of the 42 measured lines is treated as an independent determination, yields  $\pm 1.9$  km/sec. Because of the differences between the results from the different hydrogen lines and

TABLE 2  
CORRECTIONS

Telescope	Line	No. of Plates	Correction To Reduce to Lick System (Km/Sec)	P.e. (Km/Sec)
60-inch . . . . .	$H\delta$	11	$-1.3$	1.4
	$H\gamma$	19	$+5.3$	1.2
	$H\beta$	7	$+1.8$	4.8
100-inch . . . . .	$H\delta$	14	$+6.1$	1.4
	$H\gamma$	17	$+5.4$	1.2
	$H\beta$	13	$+0.4$	2.8

TABLE 3  
RESULTS OF MEASUREMENTS BEFORE AND AFTER CORRECTION

Telescope	Line	Weight	Uncorrected Displacement (Km/Sec)	Corrected Displacement (Km/Sec)
60-inch . . . . .	$H\delta$	1.2	$-7$	$-8$
	$H\gamma$	5.2	$-26$	$-21$
	$H\beta$	2.2	$-31$	$-29$
			$-26$ (weighted mean)	$-21$ (weighted mean)
100-inch . . . . .	$H\delta$	1.5	$-15$	$-9$
	$H\gamma$	6.0	$-25$	$-20$
	$H\beta$	2.7	$-22$	$-22$
			$-22$ (weighted mean)	$-19$ (weighted mean)

because of the unexpectedly large systematic corrections,  $\pm 4$  km/sec is considered as a more realistic estimate.

After these reductions had been completed, Dr. J. L. Greenstein made available to me three plates of this star exposed at Mount Palomar with a dispersion of 38 Å/mm and one with a dispersion of 18 Å/mm. Only  $H\gamma$  could be measured. The mean apparent velocity from these plates is  $-23$  km/sec, while the rough value by Münch, quoted above, is  $-20$  km/sec. The inclusion of these values would not alter the adopted velocity.

The radial velocity of 40 Eri A in the Lick system is  $-42$  km/sec, so that the apparent difference in velocity, B *minus* A, for a mean epoch of 1951 is  $+21$  km/sec, with an estimated uncertainty of  $\pm 4$  km/sec. The corresponding value of  $\Delta\lambda/\lambda$  is  $7 \times 10^{-5}$ , where  $\Delta\lambda$  is the red shift in wave-length units.

## DISCUSSION

Known effects which could give rise to an apparent velocity difference between 40 Eri B and 40 Eri A are (1) relative motion, (2) mass motions in the atmosphere of A or B, (3) asymmetrical line-absorption coefficient, and (4) gravitational red shift. Since star A is a normal dwarf and star B a white dwarf, we shall assume that effect 2 is not present. The others will be considered in turn.

From the orbit of 40 Eri BC published by van den Bos (1926) and the similar orbit by Artukhina (1948), the radial-velocity amplitude of B about the center of mass of BC is 1.3 km/sec. At the epoch of the spectrograms the radial component of B was 0.4 km/sec, the sign being unknown. This motion is so small relative to both the observed red shift and its uncertainty that it can be neglected. The parallax ( $0''.20$ ) and proper motion ( $\mu_\alpha \cos \delta : 2''.2$ ;  $\mu_\delta : 3''.4$ ) of component A of 40 Eri are known to be equal to those of BC to within small fractions of the values themselves, so that it is certain that A is a member of the same system as B and C. Van den Bos (1926) has analyzed the double-star measures of B relative to A and finds, after correcting for orbital motion of B in the system BC, a relative motion of  $6''$  per century approximately normal to the line joining stars A and B. At a distance of 5 parsecs this motion corresponds to a tangential speed of 1.4 km/sec. There is no way of knowing whether a radial component of a similar order of magnitude may be present. The relative velocity of escape is about 2.5 km/sec. We shall assume that this component of radial motion is also small relative to the measured red shift. It would have been desirable to determine the radial velocity of 40 Eri C on the Lick system as a further check. Because of the faintness of this component, however, this check is not practicable with the available equipment.

Since 40 Eri A is a normal K dwarf and since K dwarfs show no unexplained systematic residual velocity on the Lick system, we can assume that the apparent velocity difference between stars A and B results entirely from effects 3 and 4 in B. The principal causes of line broadening of hydrogen lines in stellar atmospheres are rotation and interatomic Stark effect. The former has no unsymmetrical effects. For very large electric fields, the second-order Stark effect, which is unsymmetrical, can become appreciable.<sup>2</sup> Theory and experiment agree (Rausch von Traubenbergs and Gebauer 1929) that the longward shift in  $H\gamma$  from this cause is approximately

$$\Delta\lambda = \frac{3E^2}{10^{11}} A,$$

where  $E$  is the field strength in volts per centimeter. The average effective field experienced by an atom depends on the density of perturbing ions. An estimate of the ion density may be obtained by application of the Inglis-Teller relation in the case of a spectrum with strong hydrogen lines. Combining the relevant formulas (e.g., Aller 1953, pp. 308–317) gives, for the longward shift in  $H\gamma$ ,

$$\log \Delta\lambda = 7.64 - 10 \log n_m,$$

where  $n_m$  is the quantum number of the upper energy level of the highest observable member of the Balmer series. The numerical constant has not been evaluated for lines other than  $H\gamma$ , since most of the weight of the observations is in this line. The value of  $n_m$  in the spectrum of 40 Eri B is at least 9 (Kuiper 1941). Through the co-operation of Dr. H. W. Babcock, I was able to make a tracing of his 10 A/mm plate of 40 Eri B (Babcock 1948). A faint trace of  $H10$  is visible. Hence  $n_m$  is greater than 9, and  $\Delta\lambda$  is less than  $0.012 A$ , which corresponds to a red shift of 0.8 km/sec. Moreover, the cores of the  $H$  lines, which were measured, must be produced at lower densities than the wings, the

<sup>2</sup> I am indebted to Dr. L. Spitzer for calling my attention to this fact.



overlapping of which determines  $n_m$ . We conclude that the shift resulting from asymmetrical line broadening can be neglected.

Thus gravitation appears to be the only known effect which can produce the observed red shift. It remains to compare the observed value with that predicted by general relativity, which is  $0.635 \mathcal{M}/R$  km/sec.  $\mathcal{M}$  and  $R$  are the mass and radius of the star, respectively, in solar units. The available data for  $\mathcal{M}$  and  $R$  are discussed briefly in the next paragraphs.

Artukhina (1948) has corrected van den Bos's (1926) orbit of 40 Eri BC on the basis of a few additional observations and a reconsideration of the assigned weights. Although her elements differ but little from those of van den Bos, a definitive orbit will not be obtained until a considerably greater fraction of the period, 250 years, has been covered by observations. Artukhina's elements, combined with the most recent value of the parallax of the system,  $0''.201 \pm 0''.002$  (Lippencott 1952) give  $0.63 \pm 0.05$  for the combined mass of B and C. Van den Bos gives, for the mass ratio,

$$\frac{\mathcal{M}_c}{\mathcal{M}_B + \mathcal{M}_C} = 0.31 \pm 0.02.$$

This value is based on two completely independent determinations, one (0.32) obtained from the double-star measures of B with respect to A, and the other (0.29) from parallax plates (Alden 1925). We adopt for the mass of 40 Eri B, in solar units,  $0.43 \pm 0.04$ . Since the probable error is obtained only from the internal consistency of the observations, a more realistic value would be somewhat greater than  $\pm 0.04$ .

The radius of 40 Eri B must be evaluated from the apparent magnitude, parallax, and surface brightness of the star. The magnitude of 40 Eri B on the  $V$  system (Johnson and Morgan 1953) is 9.50 (D. L. Harris, unpublished), and the parallax is  $0''.201$ . The surface brightness has been evaluated from independent observations and by different methods by Kuiper (1941) and by Berger, Chalonge, Divan, and Fringant (1952). Kuiper evaluated the effective temperature by spectrophotometry of the continuous spectrum. His value is  $13,500^\circ$ . In this case the bolometric correction must be assumed known. Berger and associates obtain the "brightness temperature" at  $\lambda 5400$  from the color temperature of the continuum, the "surface" temperature corresponding to an energy-curve defined by the intensities at the centers of the hydrogen lines, and the residual intensities of the hydrogen lines. Their value is  $13,000^\circ$ . In this case the bolometric correction is not needed. The values of the radius,  $R$ , recomputed with Harris' improved magnitude of 40 Eri B and his unpublished revision of the magnitude of the sun on the  $V$  system are, respectively, 0.017 and 0.015 solar radii. It is my opinion that before complete confidence can be placed in such determinations, a theory for the atmosphere of 40 Eri B must be developed which will predict satisfactorily the form of the continuous spectrum and the profiles of the absorption lines, including their cores. The surface brightness would emerge as a parameter, from which the radius could be calculated. For the present we adopt

$$R = 0.016 \pm 0.002 \text{ solar radii}.$$

It is well known (e.g., Kuiper 1941) that this value of the radius agrees with that predicted for a degenerate star of the mass of 40 Eri B without appreciable hydrogen (Chandrasekhar 1939; Rudkjøbing 1952). A mass of  $0.43 \pm 0.04$  corresponds to a radius of  $0.015 \pm 0.001$  (Chandrasekhar) or  $0.014 \pm 0.001$  (Rudkjøbing). Hence if 40 Eri B conforms to this theory and has no hydrogen in the interior, the radius of the sphere of degenerate matter must be close to the adopted radius of the star, 0.016.

The gravitational red shift predicted for 40 Eri B from the mass and radius adopted is  $17 \pm 3$  km/sec, the estimated uncertainty being computed from the uncertainties given for  $\mathcal{M}$  and  $R$ . This value is to be compared to the observed value of  $21 \pm 4$  km/sec. The

significance of the estimated uncertainties is not easy to assess. We conclude that the observed red shift, which we have attributed entirely to the gravitational effect, agrees with the predicted value within the uncertainties of both and, if allowance is made for all uncertainties, that the ratio of observed to predicted values must be well within the range  $\frac{1}{2}$ –2.

Walker (1951) has extended the theory of kinematic relativity into the realm of dynamics through a principle of least action. The results agree with those of general relativity for planetary motions and for the deflection of light-rays, but they predict twice as large a red shift as does general relativity. This prediction appears to be contradicted by the observations, as does the conclusion of Findlay-Freundlich (1954) concerning the magnitude of the relativity red shift.

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